Towards Parameter and Unknown Input Estimation for Multi-Rotors

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Abstract: We propose a method using observers to estimate unknown parameters and input disturbances affecting a multi-rotor system. The test bench, subject to unmeasured external disturbances, consists of a drone body actuated by two controlled motor-propellers, each influenced by its disturbances. Following an observability analysis, we design a nonlinear observer for each actuator to estimate its constant drag coefficient, disturbances, and angular velocity, using the motor's current as the measurement. We then construct a linear observer to estimate the disturbances acting on the yaw motion of the drone body, with the yaw rate as the measurement and the estimates from the actuators' observers as inputs. The drone's observer is shown to converge when the actuators' observers perform correctly. Simulation results with realistic parameters illustrate the effectiveness of the proposed approach.

Keywords: Multi-rotors, unknown input observers, parameter estimation, nonlinear systems

1. INTRODUCTION

Multi-rotor drones find widespread applications across various fields, ranging from aerial photography to civilian and military [1]. However, in real-world scenarios, drones are subject to not only known control inputs but also different types of unknown inputs and disturbances [2]. Accurately estimating these unknowns is essential for tasks such as robust controller synthesis and disturbance rejection, enabling improved reliability and performance. For instance, when the uncertainty involves the actuators' loss of effectiveness, several estimation methods have been proposed [3, 4]. In this work, we consider a test bench available at The University of Tokyo (see Figure 1) that consists of a drone body and two actuators that can be independently controlled to monitor the pitch and yaw motions [2]. Here, we simplify the problem by locking the pitch motion and focusing solely on yaw motion control. The primary goal of this work is to build a *disturbance observer* to estimate:

- Unknown parameters and disturbances acting on each actuator;
- Disturbances (e.g., wind forces, physical interactions) affecting the drone as a whole.

One of the main challenges in this task is that the dynamics of the disturbances are unknown, complicating the observer design when treating the uncertainties as additional states. To handle this, the *descriptor* system modeling [5] has been shown to be effective for such problems. It treats unknown inputs as algebraic variables rather than dynamical ones, bypassing the need to know their exact dynamics. This technique has been used in works such as [6] in the fault estimation context. However, descriptor modeling is generally limited to systems with linear or linear-like structures and relies on specific rank conditions associated with the system's dynamics and output matrices. An alternative approach, and the one adopted in this work, is to assume that the disturbances are *slow-varying* or piecewise constant. Under this assumption, disturbances can be modeled as additional states with zero derivatives [7], provided they are detectable from the extended system's outputs. This approach has been explored in studies such as [8, 9]. While straightforward and computationally efficient, the assumption of slow-varying disturbances may not always hold, particularly for fast-varying inputs. Nevertheless, in this work, the piecewise constant assumption is justified as a reasonable simplification. We perform an observability analysis to demonstrate that, under suitable conditions, the parameters and disturbances to be estimated are differentially observable from the system outputs. This implies that any abrupt changes in these unknowns are immediately detected, allowing the observer to quickly correct its estimates. To achieve this, we propose a modular observer scheme involving three observers: one for each of the two actuators and one for the drone body. The drone's observer uses the estimates from the actuators' observers as inputs, enabling a hierarchical estimation strategy. In particular, this paper uses the angular velocities from the actuators' observers to precisely estimate the yaw moment input to the drone's observer. This is an advanced and novel idea in comparison with the traditional approaches, which merely provide the drone's observer with the yaw moment command (i.e., from a motion controller). Transparently, there always exists a difference between the real and the command values, and this might degrade the accuracy of unknown input observers at the drone level. Our approach is validated through numerical simulations. Note that a passivity-based estimation scheme has been investigated for a simplified model of the actuator (neglecting the motor inductance) in [10]. Thus, our proposed scheme extends this work to include a more comprehensive model, incorporating additional system dynamics. Note that although the method has been developed and evaluated using a 2-propeller system, it can be

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straightforwardly extended to any N-propeller system. This is due to the fact that each global/local observer can be designed independently.

2. PROBLEM FORMULATION

Consider a half-quadrotor test bench propelled by two identical motor-propeller actuators, as illustrated in Figure 1. The dynamical model of each actuator, indexed by $q \in \{1, 2\}$, reads:

$$L\frac{di_q}{dt} = -Ri_q - K_e\omega_q + V_q, \tag{1a}$$

$$J_l\frac{d\omega_q}{dt} = K_m i_q - D_l\omega_q|\omega_q| - \xi_{l,q}, \tag{1b}$$

where i_q represents the current and ω_q is the angular velocity of motor q, and $\xi_{l,q}$ denotes the unknown disturbance acting on it. We propose to use the motor current as the measurement

$$y_q = i_q. \tag{1c}$$

The parameters are identical for the two actuators and are presented in Table 1. The constant drag coefficient D_l is considered unknown and is to be estimated in this paper.

Remark 1. For system (1), note that it is possible to install an encoder to measure the propeller speed ω_q. However, this significantly increases the system cost as well as the actuator inertia. Therefore, we only use the current measurement as in (1c). Note also that the controllers V_q are designed independently to drive the drone's angular position and velocity to a suitable reference trajectory, following works such as [2, 11, 12]. In this work, we assume these controllers are available and disregard any possible dependence of the control inputs on the state, treating them as exogenous inputs for observer design. Future work will include feeding back the estimates for more efficient control strategies.

The dynamical model of the drone body propelled by the two actuators reads:

$$J_g\frac{d\omega}{dt} = -D_g\omega + N_g(\omega_1, \omega_2) - \xi_g, \tag{2a}$$

where

$$N_g(\omega_1, \omega_2) = S_g(\omega_1|\omega_1| + \omega_2|\omega_2|), \tag{2b}$$

and the measurement is given by

$$y = \omega. \tag{2c}$$

The parameters for the drone model are presented in Table 1.

Remark 2. In general, there are also disturbances to the current dynamics (1a) and the disturbance/loss of effectiveness in (2b), characterized by a coefficient in $[0, 1]$. This preliminary work only considers the disturbances in (1b) and (2a).

Our objective in this paper is stated in Problem 1.

Problem 1. Design an algorithm that estimates $(i_q, \omega_q, D_l, \xi_{l,q})$ of system (1) (for each $q \in \{1, 2\}$), as well as (ω, ξ_g) of system (2), from the available measurements.

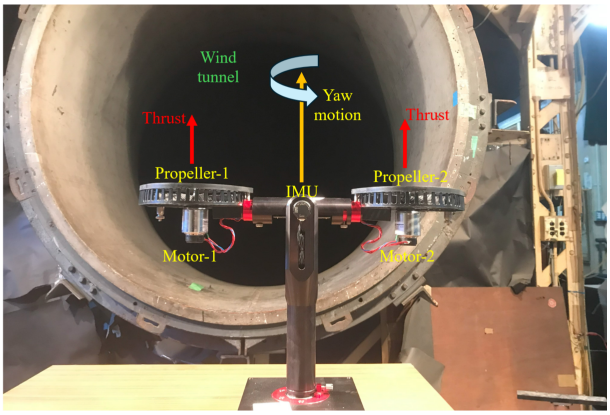

Fig. 1 Test bench at the University of Tokyo: Dual motor-propeller system with the wind tunnel to generate the external disturbance.

3. OBSERVER DESIGN FOR THE PROPELLED DRONE

3.1. Overall Strategy

To address Problem 1, a first naive approach is to concatenate (1) (two times for both actuators) with (2) into a single system and attempt to build an observer for this. However, this would be impractical given the nonlinearity and increased dimensions. Recognizing the difficulty of estimating all parameters and disturbances jointly, especially when the actuators interact with the drone through inputs that can be estimated independently (the velocities ω_q), we propose the approach illustrated in Figure 2, which is explained as follows. We construct one observer for each actuator, which also estimates ω_q, and one observer for the drone that sees ω_q as an input. Considering the ω_q's as inputs means that if these are not effectively corrected by the actuators' observers, then the drone's observer cannot work properly due to input mismatch. However, by feeding the estimates $\hat{\omega}_q$ to the drone's observer which can be shown to exhibit input-to-state stability (ISS) with respect to the unknown ω_q's, we manage to correct this one asymptotically. Each observer will also serve to estimate the corresponding unknown disturbance. With this in mind, we transform Problem 1 into Problem 2 as follows.

Problem 2. Design:

- For each $q \in \{1, 2\}$, an observer that estimates $(i_q, \omega_q, D_l, \xi_{l,q})$ of system (1);
- An observer that estimates (ω, ξ_g) of system (2), treating (ω_1, ω_2) as known inputs.

To facilitate the design, we adopt the following common assumption on the disturbances, frequently used in unknown input estimation schemes [7, 9].

Assumption 1. The disturbances $\xi_{l,q}$, $q \in \{1, 2\}$, and ξ_g are piecewise constant, i.e., $\dot{\xi}_{l,q} = 0$ and $\dot{\xi}_g = 0$ over intervals and with possible jumps at the intervals' boundaries.

Remark 3. The piecewise constant disturbances can of course make jumps at isolated instants. However, our

Table 1 Parameters of our test bench at The University of Tokyo (see Figure 1) [2]. Except for D_l which is assumed, the other parameters are either provided by the motor production company or obtained using system identification.

Parameter	Description	Value	Unit
L	actuator inductance	$1.16 \cdot 10^{-3}$	H
R	actuator resistance	8.4	Ω
J_l	actuator inertia	$4.40304 \cdot 10^{-5}$	kg·m^2
K_e	coupling coefficient	0.042	Vs/rad
K_m	coupling coefficient	0.042	N·m/A
D_l	drag coefficient	$8 \cdot 10^{-8}$	N·s^2/(m·rad^2)
J_g	drone inertia	0.022	N·m·s/rad^2
D_g	damping coefficient	0.022	N·m·s/rad
S_g	actuating coefficient	$5 \cdot 10^{-6}$	N·s^2/(m·rad^2)

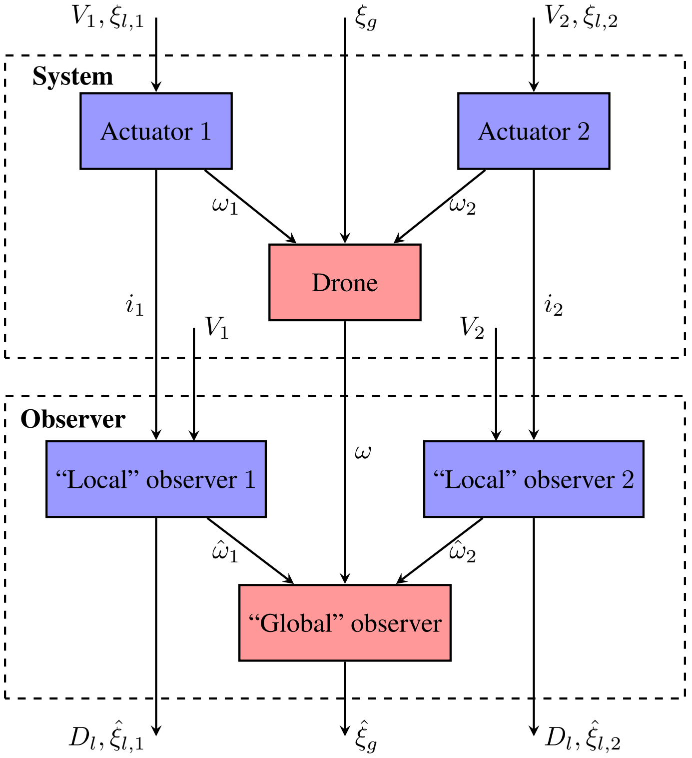

Fig. 2 General scheme of observer design.

analysis later shows that these are differentially (or instantaneously) observable from the measurements. This means that any abrupt change in these disturbances is immediately visible in the output and can then be corrected by the (arbitrarily fast) observers.

In the following sections, we detail the design of the observers outlined here.

3.2. Observer for Each Actuator

In this part, we design the observers for the actuators. We start by analyzing the observability of system (1), then propose suitable observers.

3.2.1. Observability Analysis

We begin by making the following assumptions for each actuator.

Assumption 2. For each actuator described by system (1), i.e., for each $q \in \{1, 2\}$, we assume that:

- The considered trajectories $t \mapsto (i_q(t), \omega_q(t))$, initialized in some bounded set and subject to considered bounded inputs $t \mapsto (V_q(t), \xi_{l,q}(t))$, remain in bounded sets at all times;
- The condition $\omega_q(t)\dot{\omega}_q(t) \neq 0$ holds for (almost) all $t \geq 0$.

In Assumption 2, the first item is generally satisfied given the physical system. The second item, which we later show to guarantee observability conditions, can be satisfied if each actuator rotates and undergoes accelerated/decelerated (e.g., at the lifting phase). It is also related to the notion of *persistence of excitation* usually exploited in parameter estimation (see for instance [13, 14]). Lemma 1 then states the observability of system (1).

Lemma 1. Under Assumption 1 and Assumption 2, for each $q \in \{1, 2\}$, $(i_q, \omega_q, D_l, \xi_{l,q})$ of system (1) are differentially observable, i.e., these variables are uniquely determined from (y_q, V_q) and their time derivatives up to a finite order.

Proof: The proof is the same for $q \in \{1, 2\}$. We first get that $i_q = y_q$ and from (1a), $\omega_q = \frac{1}{K_e}\left(-Ri_q - L\frac{di_q}{dt} + V_q\right) = \frac{1}{K_e}\left(-Ry_q - L\dot{y}_q + V_q\right)$, so that these are differentially observable. We also have by derivating (1b), since D_l and $\xi_{l,q}$ are both constant,

$$\frac{J_l}{K_e}(-R\dot{y}_q - L\ddot{y}_q + \dot{V}_q) = K_m y_q - D_l\omega_q|\omega_q| - \xi_{l,q}, \tag{3a}$$

$$\frac{J_l}{K_e}(-R\ddot{y}_q - L\dddot{y}_q + \ddot{V}_q) = K_m\dot{y}_q - 2D_l\dot{\omega}_q|\omega_q|. \tag{3b}$$

Note that the time derivative of the term $\omega_q|\omega_q|$, which is $2\dot{\omega}_q|\omega_q|$, is only defined when $\omega_q \neq 0$, due to the non-differentiability of the absolute value function at zero. Under Assumption 2, we get from (3b) that $D_l = \frac{K_m\dot{y}_q - \frac{J_l}{K_e}\left(-R\ddot{y}_q - L\dddot{y}_q + \ddot{V}_q\right)}{2\dot{\omega}_q|\omega_q|}$ and then from (3a) that $\xi_{l,q} = K_m y_q - \frac{J_l}{K_e}(-R\dot{y}_q - L\ddot{y}_q + \dot{V}_q) - \frac{K_m\dot{y}_q - \frac{J_l}{K_e}(-R\ddot{y}_q - L\dddot{y}_q + \ddot{V}_q)}{2\dot{\omega}_q|\omega_q|}\omega_q|\omega_q|$. Since $(\omega_q, \dot{\omega}_q)$ are expressible in (y_q, V_q) and their time derivatives, this concludes the proof. ∎

With Lemma 1 in mind, we know that we can construct for system (1) a high-gain observer of dimension four, which works when the angular velocity and acceleration are non-zero, such as in the transient phase when the drone is being lifted up. This will however face a problem whenever we enter the steady state, when ω_q has been driven to a constant reference making $\dot{\omega}_q$ near zero. However, another observation is that D_l is constant and can be well estimated in the transient phase when it is observable; on the other hand, if we know D_l, $\xi_{l,q}$ can change as a step function of time and is still observable even when $\dot{\omega}_q = 0$. Therefore, we propose to build a sequence of observers as an observer for system (1). In the transient phase, we use a four-dimensional high-gain observer that estimates D_l very fast, and we switch to using a three-dimensional observer with D_l sufficiently well estimated by the first one. These ideas will be presented in the next part.

3.2.2. Observer Design for System (1)

Under Assumption 2, we propose the nonlinear change of coordinates

$$z_1 = i_q, \tag{4a}$$

$$z_2 = -\frac{K_e}{L}\omega_q, \tag{4b}$$

$$z_3 = \frac{K_e}{LJ_l}(D_l\omega_q|\omega_q| + \xi_{l,q}), \tag{4c}$$

$$z_4 = \frac{2K_e}{LJ_l^2}D_l|\omega_q|(K_m i_q - D_l\omega_q|\omega_q| - \xi_{l,q}), \tag{4d}$$

whose inverse is

$$i_q = z_1, \tag{5a}$$

$$\omega_q = -\frac{L}{K_e}z_2, \tag{5b}$$

$$D_l = \frac{J_l^2 z_4}{2|z_2|\left(K_m z_1 - \frac{LJ_l}{K_e}z_3\right)}, \tag{5c}$$

$$\xi_{l,q} = \frac{LJ_l}{K_e}z_3 + \frac{L^2J_l^2 z_2 z_4}{2K_e^2\left(K_m z_1 - \frac{LJ_l}{K_e}z_3\right)}. \tag{5d}$$

Note that $(D_l, \xi_{l,q})$ are defined only when

$$|z_2|\left(K_m z_1 - \frac{LJ_l}{K_e}z_3\right) \neq 0,$$

which translates to $|\omega_q|\dot{\omega}_q \neq 0$ and is guaranteed by Assumption 2. This is coherent with the observability analysis in Lemma 1. With transformation (4), we obtain the z-coordinates dynamics as follows

$$\begin{aligned}
\dot{z}_1 &= \frac{1}{L}(-Ri_q - K_e\omega_q + V_q) \\
&= z_2 - \frac{R}{L}z_1 + \frac{1}{L}V_q =: z_2 + \phi_1(z_1) + \frac{1}{L}V_q,
\end{aligned} \tag{6a}$$

$$\begin{aligned}
\dot{z}_2 &= -\frac{K_e}{LJ_l}(K_m i_q - D_l\omega_q|\omega_q| - \xi_{l,q}) \\
&= z_3 - \frac{K_e K_m}{LJ_l}z_1 =: z_3 + \phi_2(z_1),
\end{aligned} \tag{6b}$$

$$\begin{aligned}
\dot{z}_3 &= \frac{2K_e}{LJ_l^2}D_l|\omega_q|(K_m i_q - D_l\omega_q|\omega_q| - \xi_{l,q}) \\
&=: z_4,
\end{aligned} \tag{6c}$$

$$\begin{aligned}
\dot{z}_4 &= \frac{2K_e}{LJ_l^2}D_l|\omega_q|\Bigg(\frac{1}{J_l\omega_q}(K_m i_q - D_l\omega_q|\omega_q| - \xi_{l,q})^2 \\
&\quad + \frac{K_m}{L}(-Ri_q - K_e\omega_q + V_q) \\
&\quad - \frac{2D_l}{J_l}|\omega_q|(K_m i_q - D_l\omega_q|\omega_q| - \xi_{l,q})\Bigg) \\
&= -\frac{2K_e|z_2|z_4}{LJ_l z_2}\left(K_m z_1 - \frac{LJ_l}{K_e}z_3\right) \\
&\quad + \frac{2|z_2|z_4}{K_m z_1 - \frac{LJ_l}{K_e}z_3}\times \\
&\quad \times\left(-\frac{K_m R}{L}z_1 + K_m z_2 - \frac{LJ_l}{K_e}z_4 + \frac{K_m}{L}V_q\right) \\
&=: \phi_4(z),
\end{aligned} \tag{6d}$$

with the output

$$y_q = z_1. \tag{6e}$$

We can then build a high-gain observer [15] for system (6), which is of triangular canonical form. Saturation of the observer maps during transient is necessary and is possible under the first item of Assumption 2. Then, the estimates of $(i_q, \omega_q, D_l, \xi_{l,q})$ are recovered using (5).

Now, with D_l known, we perform the change of coordinates from $(i_q, \omega_q, \xi_{l,q})$ into (z_1, z_2, z_3) (and not z_4) following (4), whose inverse is:

$$i_q = z_1, \tag{7a}$$

$$\omega_q = -\frac{L}{K_e} z_2, \tag{7b}$$

$$\xi_{l,q} = \frac{LJ_l}{K_e} z_3 + \frac{D_l L^2}{K_e^2} z_2 |z_2|. \tag{7c}$$

With this transformation, we obtain the z-coordinates dynamics as follows

$$\dot{z}_1 = z_2 + \phi_1(z_1) + \frac{1}{L} V_q, \tag{8a}$$

$$\dot{z}_2 = z_3 + \phi_2(z_1), \tag{8b}$$

$$\dot{z}_3 = \frac{2D_l}{J_l^2} |z_2| \left(K_m z_1 - \frac{LJ_l}{K_e} z_3 \right) =: \phi_3(z), \tag{8c}$$

with the output

$$y_q = z_1. \tag{8d}$$

This takes the triangular canonical form and we can again build for it a high-gain observer. Note that for (7), we do not need the condition in the second item of Assumption 2 to hold. Therefore, we propose to design an observer for system (6) with inverse (5) to estimate D_l. Then, since D_l is constant, we keep this value and design an observer for system (8), assuming D_l is known, with inverse (7) to estimate $\xi_{l,q}$ online.

3.3. Observer for the Drone

Now, let us design the observer for the drone (2). Because (ω_1, ω_2) are seen as inputs, we rewrite system (2) as

$$\dot{x} = Ax + B(\omega_1, \omega_2), \qquad y = Cx, \tag{9}$$

with state $x = (\omega, \xi_g)$, where $A = \begin{pmatrix} -\frac{D_g}{J_g} & -\frac{1}{J_g} \\ 0 & 0 \end{pmatrix}$, $B(\omega_1, \omega_2) = \begin{pmatrix} \frac{S_g}{J_g}(\omega_1|\omega_1| + \omega_2|\omega_2|) \\ 0 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 0 \end{pmatrix}$. Given the linear form and that (A, C) is observable, we can find a constant gain L_o such that $A - L_o C$ is Hurwitz. Then, a simple observer for system (2) takes the form

$$\dot{\hat{x}} = A\hat{x} + B(\hat{\omega}_1, \hat{\omega}_2) + L_o(y - C\hat{x}), \tag{10}$$

with $(\hat{\omega}_1, \hat{\omega}_2)$ estimated by the actuators' observers (see Figure 2). We then deduce that the estimation error $\tilde{x} := x - \hat{x}$ verifies

$$\dot{\tilde{x}} = (A - L_o C)\tilde{x} + B(\omega_1, \omega_2) - B(\hat{\omega}_1, \hat{\omega}_2). \tag{11}$$

Combining the Hurwitzness of $A - L_o C$ with the fact that $(\omega_1(t) - \hat{\omega}_1(t), \omega_2(t) - \hat{\omega}_2(t)) \to 0$ as $t \to +\infty$ thanks to the actuators' observers, we get that $\tilde{x}(t) \to 0$ as $t \to +\infty$. Note that because the problem is linear, unlike in Section 3.2, here we do not need to assume that the state x remains in a bounded set.

4. SIMULATIONS

In this section, we provide simulation scenarios and results to illustrate our methods. First, we propose a simple experiment that can be performed offline to estimate the unknown parameter D_l, by designing a simple Proportional-Integral-Derivative (PID) control input V_q that steers the current i_q to a sinusoidal reference. This controller keeps the velocity ω_q from converging to a settling value, though its derivative crosses zero from time to time. Simulations in Figure 3 show effective convergence outside of the non-observable instants (when $\dot{\omega}_q = 0$). Since D_l is constant, we can estimate this unknown parameter.

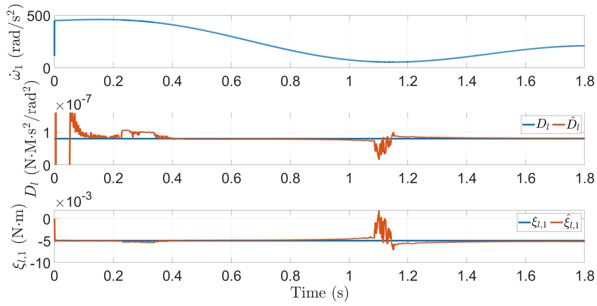

Fig. 3 Estimation of $(i_1, \omega_1, D_l, \xi_{l,1})$ in an actuator (1) using a four-dimensional observer.

With D_l known, simulations are performed where $(i_q, \omega_q, \xi_{l,q})$ for each actuator are estimated using a three-dimensional high-gain observer. We consider $\xi_{l,q}$ a varying step function of time. The inputs (V_1, V_2) are simple PID controllers that serve to drive the drone's angular position to a sinusoidal reference. From Figure 4, we see that each time the disturbance changes, the observer notices this change and corrects it immediately, thanks to differential observability. Note again that this observability does not require ω (or (ω_1, ω_2)) to satisfy any condition.

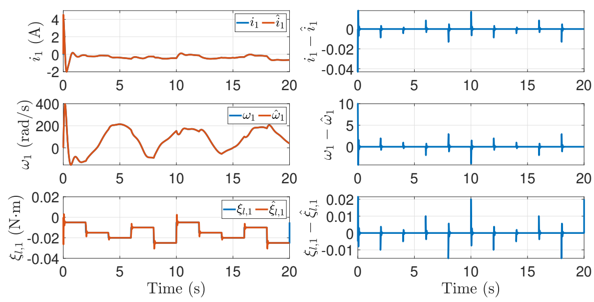

Fig. 4 Estimation of $(i_1, \omega_1, \xi_{l,1})$ in an actuator (1) using a three-dimensional observer (with D_l estimated separately).

Last, we estimate the disturbance ξ_g in the test bench using observer (10), with $(\hat{\omega}_1, \hat{\omega}_2)$ provided by the actuators' observers. Because $(\hat{\omega}_1, \hat{\omega}_2)$ come from the actuators' observers, we make an error whenever a new disturbance affects either motor. However, as seen in Figure 5, the estimates will then be corrected immediately by the observers.

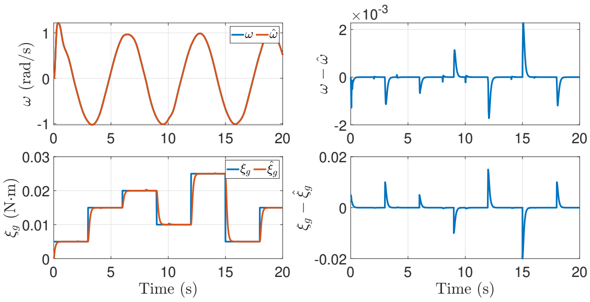

Fig. 5 Estimation of (ω, ξ_g) in the test bench (2) using a simple observer (with (ω_1, ω_2) estimated separately and treated as inputs).

5. CONCLUSION

In this paper, we present observer designs for estimating unknown parameters and disturbances in a test bench system comprising a drone body and two actuators. Through observability analysis, we demonstrate that these unknowns are differentially observable, potentially under specific conditions. To achieve this, we design high-gain observers for each actuator and a simple linear observer for the drone body, which utilizes the estimates from the actuator observers as inputs. The proposed methods are validated through numerical simulations.

Future work will focus on extending the estimation framework to address other types of parameters and disturbances within this system. Additionally, we plan to design feedback controllers that leverage these estimates to enhance the system's performance and robustness. Experiments on the test bench will also be performed.

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