Feedforward with Acceleration and Snap using Sampled-Data Differentiator for a Multi-Modal Motion System

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Abstract: Sampled-data control requires both on-sample and intersample performance in high-precision mechatronic systems. The aim is to design a discrete-time linearly parameterized feedforward controller to improve both on-sample and intersample performance in a multi-modal motion system. The continuous-time performance is considered as state compatibility by a multirate zero-order-hold differentiator. The developed approach enables the linearly parameterized feedforward controller design for sampled-data systems with physically intuitive tuning parameters. The performance improvement is validated by comparing the developed approach with a conventional approach using a backward differentiator for a multi-modal motion system.

Keywords: feedforward, discrete-time system, sampled-data control, zero-order-hold, multirate

1. INTRODUCTION

Feedforward control is essential for reference tracking in industrial high-precision mechatronic systems such as semiconductor lithography systems (Steinbuch et al., 2021) and high-speed scanners (Ito et al., 2019). The feedforward controllers are usually implemented in digital hardware, and the parameter of the feedforward controller is first designed from the model of the controlled system, and secondly tuned by experimental data. For intuitive tuning of the feedforward controller, it is preferable that the feedforward controller is represented by the parameters with physical meaning.

The linearly parameterized feedforward control (Lambrechts et al., 2005) has an advantage because the tuning process is physically intuitive. Model inversion based feedforward controllers such as zero phase error tracking control (Tomizuka and Sun, 2021) are widely used to improve tracking performance. However, it is time-consuming to identify the model of the controlled system and hard to tune parameters manually.

The feedforward controller design for higher-order motion systems has a challenge because of the model complexity and it results in many non-intuitive parameters in the feedforward controller. Industrial mechatronic systems are modeled as the dominant rigid mode at a lower frequency and several flexible modes at a higher frequency due to limited mechanical stiffness (Gawronski, 2004). The feedforward controller can be parameterized physically intuitive using the modal characteristics.

The sampled-data characteristics should be considered in the feedforward controller design because of the limitation of the sampling frequency. The feedforward controllers are usually implemented in discrete-time with sampler and zero-order-hold (Chen and Francis, 1995). Several related studies using the multirate feedforward control (Fujimoto et al., 2001) are developed to improve intersample performance by compensating for the oscillation of the Nyquist frequency.

Although several linearly parameterized feedforward control approaches exist, the on-sample performance is mainly discussed in conventional approaches and the sampled-data characteristics with sampler and zero-order-hold are not considered. The conventional linearly parameterized feedforward control is designed by using the backward differentiator (Lambrechts et al., 2005) and it is not compatible with the states of the continuous-time motion system. The pre-existing state tracking approaches for both on-sample and intersample performance (van Zundert et al., 2020) need the model of the controlled system based on the system identification and they are not linearly parameterized in tuning parameters.

The main contribution of this paper is the linearly parameterized feedforward control approach considering sampled-data characteristics to improve both on-sample and intersample performance in multi-modal motion systems. The contributions in this paper are the following:

Contribution 1. The multirate zero-order-hold differentiator is developed to design the discrete-time basis functions $\boldsymbol{\Psi}[k]$ that satisfy state compatibility for the continuous-time reference $r(t)$.

Contribution 2. The linearly parameterized feedforward considering sampled-data characteristics is designed with a multirate zero-order-hold differentiator and both on-sample and intersample performance improvement is experimentally validated in a multi-modal motion system.

The outline is as follows. In Section 2, the problem that is considered in this paper is formulated. In Section 3, the design method of the feedforward controller using the multirate zero-order-hold differentiator is developed, constituting Contribution 1. In Section 4, the advantage of the approach is demonstrated in the experiment with a multi-modal motion system, constituting Contribution 2. In Section 5, conclusions are presented.

2. PROBLEM FORMULATION

In this section, the problem to improve continuous-time tracking performance is formulated. First, the requirements in this paper are described. Second, the reference tracking problem is defined in intersample performance. Third, the low-order feedforward control approach is investigated for reference tracking in a multi-modal motion system. Fourth, the low-order feedforward control approach is implemented in discrete-time. Finally, the problem in the conventional approach is described.

2.1 Problem description

In this paper, the feedforward controller is designed with respect to the following requirements:

Requirement 1. The feedforward controller is linearly parameterized with physical parameters for intuitive tuning.

Requirement 2. The sampled-data characteristics with sampler and zero-order-hold are considered in the feedforward controller design.

Requirement 3. The designed feedforward controller can be applied to multi-modal motion systems.

To improve both on-sample and intersample performance, the main problem in the linearly parameterized feedforward controller is the discrete-time basis function design that is compatible with the continuous-time reference $r(t)$ considering sampled-data characteristics.

2.2 Reference tracking for intersample performance

The considered tracking control configuration is shown in Fig. 1, with reference $r(t) \in \mathbb{R}$, control input $u(t) \in \mathbb{R}$, and output $y(t) \in \mathbb{R}$. The continuous-time linear time-invariant system G is controlled by the sampled-data controller that consists of feedforward controller $F(\theta)$, feedback controller K, sampler $\mathcal{S}$, and zero-order-hold $\mathcal{H}$, where sampler and zero-order-hold are defined as follows:

Definition 1. (Sampler). The sampler $\mathcal{S}$ with sampling time T_s is defined as

$$\mathcal{S} : r(t) \mapsto r[k], \quad r[k] = r(kT_s). \tag{1}$$

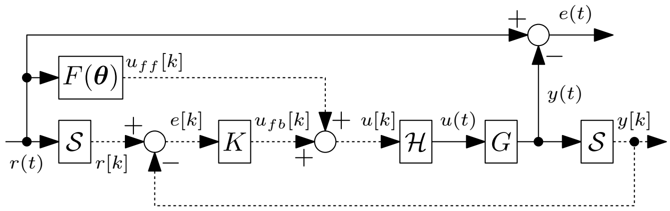

Fig. 1. Tracking control diagram. The continuous-time system G is controlled by the feedforward controller $F(\boldsymbol{\theta})$ and the discrete-time feedback controller K with sampler $\mathcal{S}$ and zero-order-hold $\mathcal{H}$. The objective is to minimize the continuous-time error $e(t)$. The solid and dotted lines denote the continuous-time and discrete-time signals, respectively.

Definition 2. (Zero-order-hold). The zero-order-hold $\mathcal{H}$ with sampling time T_s is defined as

$$\mathcal{H} : u[k] \mapsto u(t), \quad u(kT_s + \tau) = u[k], \quad \tau = [0, T_s). \tag{2}$$

The control objective in this paper is to minimize the continuous-time error $e(t)$. Traditionally, the conventional discrete-time controller only focuses on the on-sample performance with the discrete-time error $e[k]$. To improve the continuous-time error $e(t)$, both on-sample and intersample performance should be considered.

2.3 Low-order feedforward for multi-modal motion system

The goal of the feedforward controller design is to extend the rigid mode behavior over a frequency range as high as possible. Note that the controlled system and the controllers are assumed to be the continuous-time system only in this subsection.

Industrial mechatronic systems consist of the dominant rigid mode at a lower frequency and several flexible modes at a higher frequency due to limited mechanical stiffness. The continuous-time single-input single-output multi-modal motion system is defined as

$$G_c(s) = \frac{1}{ms^2} + \sum_{i=1}^{n_{fl}} \frac{k_i}{m(s^2 + 2\zeta_i\omega_i s + \omega_i^2)}, \tag{3}$$

where m is the total mass of the system, n_{fl} is the number of the flexible modes. The resonance frequency, the damping coefficient, and the mode gain at the i^{th} mode are ω_i, ζ_i, and $k_i \in \{-1, 1\}$, respectively.

To compensate for both rigid mode and flexible modes, the traditional acceleration feedforward is extended with the additional snap feedforward (Boerlage et al., 2004), and the ideal feedforward controller $F^*(s)$ is defined as

$$F^*(s) = ms^2 + D^*(s)s^4, \tag{4}$$

where $D^*(s)$ is the coefficient of the snap feedforward.

The objective of the feedforward $F^*(s)$ is to minimize the closed-loop error given by

$$e(s) = S(s)r(s) - S(s)G_c(s)F^*(s)r(s), \tag{5}$$

where $S(s)$ denotes the sensitivity function and is defined as

$$S(s) = (1 + G_c(s)K(s))^{-1}. \tag{6}$$

It results in $F^*(s) = G_c^{-1}(s)$ and $D^*(s)$ is given by

$$D^*(s) = G_c^{-1}(s)\frac{1}{s^4} - m\frac{1}{s^2}. \tag{7}$$

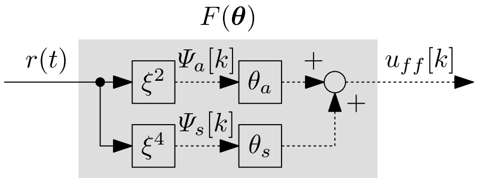

Fig. 2. Linearly parameterized feedforward with acceleration and snap.

Assuming the reference trajectory in the mechatronic systems mainly contains the low-frequency components, the low-frequency contribution of the snap feedforward is given by

$$D = \lim_{s\to 0} D^*(s) = \lim_{s\to 0}\left(G_c^{-1}(s)\frac{1}{s^4} - m\frac{1}{s^2}\right) = \frac{-m\sum_{i=1}^{n_{fl}} k_i \prod_{j\neq i}\omega_j^2}{\prod_{i=1}^{n_{fl}}\omega_i^2}. \tag{8}$$

Finally, the low-order feedforward controller with acceleration and snap is given by

$$F(s) = ms^2 + Ds^4, \tag{9}$$

where m and D are the tuning parameters in acceleration and snap.

2.4 Feedforward implementation with basis functions

The feedforward controller design method is discussed above in continuous-time. However, the controllers are implemented to digital hardware in discrete-time. As a result, the continuous-time differentiator s used in the feedforward controller is conventionally replaced by ξ which consists of the discrete-time differentiator and sampler $\mathcal{S}$.

The linearly parameterized feedforward with acceleration and snap is shown in Fig. 2. The feedforward controller $F(\boldsymbol{\theta})$ from the continuous-time reference $r(t)$ to design the discrete-time feedforward input $u_{ff}[k]$ is defined as

$$F(\boldsymbol{\theta}) = \begin{bmatrix}\xi^2 & \xi^4\end{bmatrix}\begin{bmatrix}\theta_a \\ \theta_s\end{bmatrix}, \tag{10}$$

where $\boldsymbol{\theta} = [\theta_a\ \theta_s]^\top$ are the tuning parameters in acceleration and snap.

Finally, the discrete-time feedforward input $u_{ff}[k]$ with acceleration and snap is given by

$$u_{ff}[k] = F(\boldsymbol{\theta})r(t) = \boldsymbol{\Psi}[k]\boldsymbol{\theta} = [\Psi_a[k]\ \Psi_s[k]]\begin{bmatrix}\theta_a \\ \theta_s\end{bmatrix}, \tag{11}$$

where $\boldsymbol{\Psi}[k] = [\Psi_a[k]\ \Psi_s[k]] = \begin{bmatrix}\xi^2 & \xi^4\end{bmatrix} r(t)$ are the discrete-time basis functions that are compatible with the acceleration and snap of the continuous-time reference $r(t)$, respectively.

2.5 Feedforward with acceleration and snap using conventional backward differentiator

In the conventional approach (Lambrechts et al., 2005), the discrete-time basis functions are designed by the continuous-time reference $r(t)$ and the backward differentiator that is given by

$$\xi_{bd}^n = \left(\frac{1-z^{-1}}{T_s}\right)^n z^{\frac{n}{2}}\mathcal{S}, \tag{12}$$

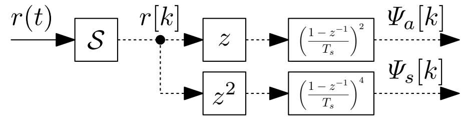

Fig. 3. Basis functions of acceleration and snap with a backward differentiator. The solid and dotted lines denote the continuous-time and discrete-time signals, respectively.

where $z = e^{sT_s}$ is the shift operator defined as $z^{-n}r[k] = r[k-n]$. It is assumed that n is even and the $z^{\frac{n}{2}}$ denotes the phase compensation. When n is odd, the phase compensation consists of the half sample shift $z^{\frac{1}{2}}$ and the approximation of averaging the current and previous value is considered (Lambrechts et al., 2005).

The basis function design in acceleration and snap with backward differentiator is shown in Fig. 3 and the basis functions are given by

$$\boldsymbol{\Psi}_{bd}[k] = \begin{bmatrix}\xi_{bd}^2 & \xi_{bd}^4\end{bmatrix} r(t) = \left[\left(\frac{1-z^{-1}}{T_s}\right)^2 zr[k] \quad \left(\frac{1-z^{-1}}{T_s}\right)^4 z^2 r[k]\right], \tag{13}$$

where $r[k] = \mathcal{S}r(t)$.

Finally, the discrete-time feedforward input $u_{ff}[k]$ with backward differentiator is given by

$$u_{ff}[k] = \boldsymbol{\Psi}_{bd}[k]\boldsymbol{\theta}, \tag{14}$$

where $\boldsymbol{\theta} = [\theta_a\ \theta_s]^\top$ is the tuning parameter.

Although the on-sample performance is mainly discussed in the conventional approach using the backward differentiator, the sampled-data characteristics with sampler and zero-order-hold are not considered.

3. LINEARLY PARAMETERIZED FEEDFORWARD CONTROLLER DESIGN USING MULTIRATE ZERO-ORDER-HOLD DIFFERENTIATOR

In this section, the linearly parameterized feedforward controller design method considering sampled-data characteristics is presented. The improvement of both on-sample and intersample performance is based on the state compatibility in a sampled-data system with zero-order-hold and integrators. The basis functions are designed by the multirate zero-order-hold differentiator. The approach is applied to the low-order feedforward controller design with acceleration and snap for multi-modal motion systems. It results in Contribution 1.

3.1 State compatibility of differentiator in sampled-data system with zero-order-hold and integrators

The continuous-time differentiator s used in the feedforward controller should be replaced by the sampled-data differentiator ξ defined as follows:

Definition 3. (Sampled-data differentiator). The n^{th} order sampled-data differentiator ξ^n with sampling time T_s is the conversion from the continuous-time signal $r(t)$ to the discrete-time signal $\Psi_n[k]$ that is compatible with the n^{th} order derivative of $r(t)$ and defined as

$$\Psi_n[k] = \xi^n r(t). \tag{15}$$

In the n samples lifted system, the exact state tracking can be achieved in every n samples using such as a minimum-time dead-beat control (Goodwin et al., 2000) and multirate feedforward control (Fujimoto et al., 2001). In such cases, the states in every n samples are given by the multirate sampler defined as follows:

Definition 4. (Multirate sampler). The multirate sampler $\mathcal{S}_n$ in every n samples with sampling time T_s is defined as

$$\mathcal{S}_n : r(t) \mapsto r[i_n], \quad r[i_n] = r(knT_s). \tag{16}$$

The state-space representation of the continuous-time n^{th} order integrator in the controllable canonical form is given by

$$\left(\frac{1}{s}\right)^n = H_{nc} \overset{s}{=} \left[\begin{array}{c|c} \boldsymbol{A}_{nc} & \boldsymbol{b}_{nc} \\ \hline \boldsymbol{c}_{nc} & 0 \end{array}\right] = \left[\begin{array}{cccc|c} 0 & 1 & 0 & 0 & 0 \\ 0 & \ddots & \ddots & 0 & \vdots \\ 0 & 0 & \ddots & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ \hline 1 & 0 & \cdots & 0 & 0 \end{array}\right], \tag{17}$$

where $\boldsymbol{A}_{nc} \in \mathbb{R}^{n\times n}$, $\boldsymbol{b}_{nc} \in \mathbb{R}^{n\times 1}$, and $\boldsymbol{c}_{nc} \in \mathbb{R}^{1\times n}$. To improve both on-sample and intersample performance in sampled-data systems with zero-order-hold and integrators, the basis functions used in the linearly parameterized feedforward controller should satisfy the state compatibility defined as follows:

Definition 5. (State compatibility). The discrete-time signal $\Psi_n[k]$ compatible with the n^{th} order derivative signal of the continuous-time signal $r(t)$ satisfies state compatibility if the output through the system consisted of the continuous-time $(n-m)^{\text{th}}$ order integrator $H_{(n-m)c}$ and zero-order-hold $\mathcal{H}$ is equal to the continuous-time m^{th} order derivative signal of $r(t)$ in every n samples sampled by multirate sampler $\mathcal{S}_n$ and defined as

$$\mathcal{S}_n \frac{\mathrm{d}^m}{\mathrm{d}t^m} r(t) = \mathcal{S}_n H_{(n-m)c} \mathcal{H} \Psi_n[k], \tag{18}$$

where $m = 0, 1, \ldots, n-1$.

3.2 Multirate zero-order-hold differentiator for intersample performance

To improve the intersample performance in the discrete-time system, the states of the reference trajectory are considered. The multirate zero-order-hold differentiator is designed by the inverse of the continuous-time integrator discretized by sampler and zero-order-hold to satisfy the state compatibility. In this paper, it is assumed that the continuous-time reference $r(t)$ is class $\mathcal{C}^{n-1}$ and differentiable at least $n-1$ times.

To satisfy the n states compatibility in every n samples, the lifted signal is considered using the lifted operator defined as follows:

Definition 6. (Lifting operator). The lifting operator $\mathcal{L}_n$ in every n samples is defined as

$$\mathcal{L}_n : u[k] \mapsto \underline{u}[i_n], \tag{19}$$

where

$$\underline{u}[i_n] = [u[ni_n] \ u[ni_n+1] \ \cdots \ u[ni_n+(n-1)]]^{\mathsf{T}} \in \mathbb{R}^n. \tag{20}$$

The n sample lifted system is defined as follows:

Definition 7. (Lifted system). Consider a discrete-time system $H_d \overset{z}{=} \boldsymbol{C}_d(z\boldsymbol{I} - \boldsymbol{A}_d)^{-1}\boldsymbol{B}_d + \boldsymbol{D}_d$. The relation between the input and the output in the n sample lifted system of H_d is given by

$$\underline{y}[i_n] = \mathcal{L}_n y[k] = (\mathcal{L}_n H_d \mathcal{L}_n^{-1})(\mathcal{L}_n u[k]) = \underline{H}_d \underline{u}[i_n], \tag{21}$$

where

$$\underline{y}[i_n] = [y[ni_n] \ y[ni_n+1] \ \cdots \ y[ni_n+(n-1)]]^{\mathsf{T}} \in \mathbb{R}^n, \tag{22}$$

and the lifted system $\underline{H}_d$ is defined as

$$\underline{H}_d \overset{z^n}{=} \mathcal{L}_n H_d \mathcal{L}_n^{-1} = \left[\begin{array}{c|c} \underline{\boldsymbol{A}}_d & \underline{\boldsymbol{B}}_d \\ \hline \underline{\boldsymbol{C}}_d & \underline{\boldsymbol{D}}_d \end{array}\right]$$
$$= \left[\begin{array}{c|ccccc} \boldsymbol{A}_d^n & \boldsymbol{A}_d^{n-1}\boldsymbol{B}_d & \boldsymbol{A}_d^{n-2}\boldsymbol{B}_d & \cdots & \boldsymbol{A}_d\boldsymbol{B}_d & \boldsymbol{B}_d \\ \hline \boldsymbol{C}_d & \boldsymbol{D}_d & \boldsymbol{O} & \cdots & \cdots & \boldsymbol{O} \\ \boldsymbol{C}_d\boldsymbol{A}_d & \boldsymbol{C}_d\boldsymbol{B}_d & \boldsymbol{D}_d & \ddots & & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \boldsymbol{C}_d\boldsymbol{A}_d^{n-2} & \boldsymbol{C}_d\boldsymbol{A}_d^{n-3}\boldsymbol{B}_d & \boldsymbol{C}_d\boldsymbol{A}_d^{n-4}\boldsymbol{B}_d & \ddots & \boldsymbol{D}_d & \boldsymbol{O} \\ \boldsymbol{C}_d\boldsymbol{A}_d^{n-1} & \boldsymbol{C}_d\boldsymbol{A}_d^{n-2}\boldsymbol{B}_d & \boldsymbol{C}_d\boldsymbol{A}_d^{n-3}\boldsymbol{B}_d & \cdots & \boldsymbol{C}_d\boldsymbol{B}_d & \boldsymbol{D}_d \end{array}\right]. \tag{23}$$

Considering the states in discrete-time, the n^{th} order integrator that is discretized by sampler and zero-order-hold is given by

$$H_{nd} \overset{z}{=} \mathcal{S} H_{nc} \mathcal{H} = \left[\begin{array}{c|c} \boldsymbol{A}_{nd} & \boldsymbol{b}_{nd} \\ \hline \boldsymbol{c}_{nd} & 0 \end{array}\right] = \left[\begin{array}{c|c} e^{\boldsymbol{A}_{nc}T_s} & \boldsymbol{A}_{nc}^{-1}(e^{\boldsymbol{A}_{nc}T_s} - \boldsymbol{I})\boldsymbol{b}_{nc} \\ \hline \boldsymbol{c}_{nc} & 0 \end{array}\right]. \tag{24}$$

To design the inverse of the n^{th} order integrator discretized by sampler and zero-order-hold, the n sample lifted system is given by

$$\underline{H}_{nd} \overset{z^n}{=} \mathcal{L}_n H_{nd} \mathcal{L}_n^{-1} = \left[\begin{array}{c|c} \underline{\boldsymbol{A}}_{nd} & \underline{\boldsymbol{B}}_{nd} \\ \hline \underline{\boldsymbol{C}}_{nd} & \underline{\boldsymbol{D}}_{nd} \end{array}\right], \tag{25}$$

and in state-space representation defined as

$$\boldsymbol{x}_n[i_n+1] = \underline{\boldsymbol{A}}_{nd}\boldsymbol{x}_n[i_n] + \underline{\boldsymbol{B}}_{nd}\underline{u}[i_n] \tag{26}$$
$$\underline{y}[i_n] = \underline{\boldsymbol{C}}_{nd}\boldsymbol{x}_n[i_n] + \underline{\boldsymbol{D}}_{nd}\underline{u}[i_n] \tag{27}$$

where

$$\boldsymbol{x}_n[i_n] = [x_0[i_n] \ x_1[i_n] \ \cdots \ x_{n-1}[i_n]]^{\mathsf{T}} \in \mathbb{R}^n. \tag{28}$$

Satisfying the state compatibility, the relationship between the reference and the states is given by

$$\boldsymbol{r}_n[i_n] = \boldsymbol{x}_n[i_n], \tag{29}$$

where

$$\boldsymbol{r}_n[i_n] = \mathcal{S}_n \left[1 \ \frac{\mathrm{d}}{\mathrm{d}t} \ \cdots \ \frac{\mathrm{d}^{n-1}}{\mathrm{d}t^{n-1}}\right]^{\mathsf{T}} r(t) = [r_0[i_n] \ r_1[i_n] \ \cdots \ r_{n-1}[i_n]]^{\mathsf{T}} \in \mathbb{R}^n. \tag{30}$$

From the discussions above, the multirate zero-order-hold differentiator is given as follows:

Theorem 1. (Multirate zero-order-hold differentiator.) From (26) and (29), considering the inverse of the continuous-time n^{th} order integrator discretized by sampler and zero-order-hold using the multirate feedforward control (Fujimoto et al., 2001), the n^{th} order multirate zero-order-hold differentiator that satisfies the state compatibility is given by

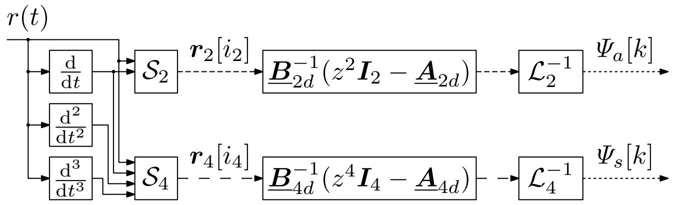

Fig. 4. Basis functions of acceleration and snap using multirate zero-order-hold differentiator. The solid line denotes the continuous-time signal. The dotted, high-frequency dashed and low-frequency dashed lines denote the discrete-time signal sampled by T_s, $2T_s$, and $4T_s$, respectively.

$$\xi_{mr}^{n} = \mathcal{L}_n^{-1} \underline{\boldsymbol{B}}_{nd}^{-1}(z^n \boldsymbol{I}_n - \underline{\boldsymbol{A}}_{nd})\mathcal{S}_n \left[1 \ \frac{\mathrm{d}}{\mathrm{d}t} \ \cdots \ \frac{\mathrm{d}^{n-1}}{\mathrm{d}t^{n-1}}\right]^{\mathsf{T}}. \tag{31}$$

Proof 1. See *Definition 5* and Fujimoto et al. (2001). □

3.3 Feedforward with acceleration and snap using multirate zero-order-hold differentiator

To design the feedforward controller with acceleration and snap, the lifted systems of the double integrator and the 4^{th} integrator discretized by zero-order-hold are given by

$$\underline{H}_{2d} \overset{z^2}{=} \mathcal{L}_2 \mathcal{S} H_{2c} \mathcal{H} \mathcal{L}_2^{-1} = \left[\begin{array}{c|c} \underline{\boldsymbol{A}}_{2d} & \underline{\boldsymbol{B}}_{2d} \\ \hline \underline{\boldsymbol{C}}_{2d} & \underline{\boldsymbol{D}}_{2d} \end{array}\right], \tag{32}$$

$$\underline{H}_{4d} \overset{z^4}{=} \mathcal{L}_4 \mathcal{S} H_{4c} \mathcal{H} \mathcal{L}_4^{-1} = \left[\begin{array}{c|c} \underline{\boldsymbol{A}}_{4d} & \underline{\boldsymbol{B}}_{4d} \\ \hline \underline{\boldsymbol{C}}_{4d} & \underline{\boldsymbol{D}}_{4d} \end{array}\right], \tag{33}$$

where the continuous-time double integrator H_{2c} and 4^{th} integrator H_{4c} are represented in controllable canonical form, respectively.

The basis function design using multirate zero-order-hold differentiator is shown in Fig. 4 and the basis functions are given by

$$\begin{aligned} \boldsymbol{\Psi}_{mr}[k] &= \left[\xi_{mr}^2 \ \xi_{mr}^4\right] r(t) \\ &= \begin{bmatrix} \mathcal{L}_2^{-1} \underline{\boldsymbol{B}}_{2d}^{-1}(z^2 \boldsymbol{I}_2 - \underline{\boldsymbol{A}}_{2d}) \boldsymbol{r}_2[i_2] \\ \mathcal{L}_4^{-1} \underline{\boldsymbol{B}}_{4d}^{-1}(z^4 \boldsymbol{I}_4 - \underline{\boldsymbol{A}}_{4d}) \boldsymbol{r}_4[i_4] \end{bmatrix}^{\mathsf{T}}. \end{aligned} \tag{34}$$

Finally, the discrete-time feedforward input $u_{ff}[k]$ using the multirate zero-order-hold differentiator is given by

$$u_{ff}[k] = \boldsymbol{\Psi}_{mr}[k]\boldsymbol{\theta}, \tag{35}$$

where $\boldsymbol{\theta} = [\theta_a \ \theta_s]^{\mathsf{T}}$ is the tuning parameter.

4. APPLICATION IN MULTI-MODAL MOTION SYSTEM

In this section, the approach in Section 3 is applied to a multi-modal motion system. The experimental results demonstrate the performance improvement in both rigid and flexible modes. It results in Contribution 2.

4.1 Setup

The experimental setup of the two-inertia system is shown in Fig. 5. The system is modeled as the multi-modal representation and given by

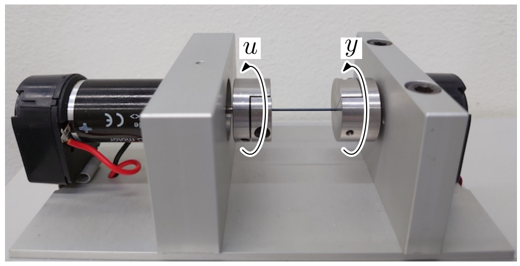

Fig. 5. Experimental setup of a two-inertia system connected via a flexible shaft. The motor on the left side is used as an input u and the encoder on the right side is used as an output y, respectively.

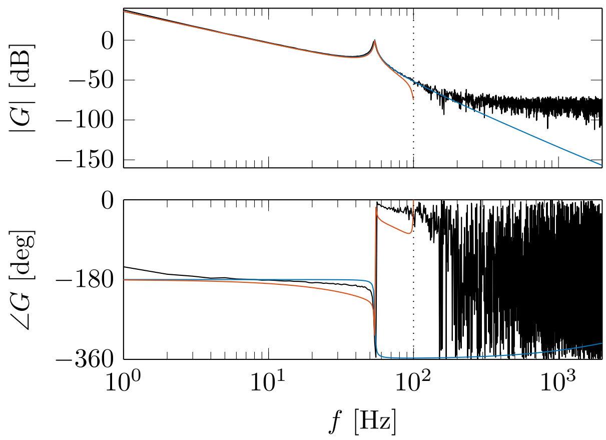

Fig. 6. Bode diagram of experimental setup: frequency response data (—), continuous-time model G_c (—), and discrete-time model G_d (—). Nyquist frequency is shown in a black dotted line (····).

$$G_c(s) = G_{rb}(s) + G_{fl}(s) \tag{36}$$

$$= \frac{1}{ms^2} + \frac{k}{m(s^2 + 2\zeta\omega s + \omega^2)}, \tag{37}$$

where $m = 0.0004$, $k = -1$, $\zeta = 0.01$, and $\omega = 2\pi \times 54$ rad/s.

The frequency response data, the continuous-time model G_c, and the discrete-time model G_d are shown in Fig. 6. Note that these models are not directly used for the feedforward controller design but only used for the physical analysis.

4.2 Conditions

The continuous-time reference $r(t)$ is the 4^{th} order polynomial trajectory shown in Fig. 7. The sampling time of the discrete-time controller is $T_s = 5\,\text{ms}$. The continuous-time output $y(t)$ is obtained by higher sampling frequency in every $250\,\mu\text{s}$ only for evaluation of the continuous-time error $e(t)$.

The approach using the multirate zero-order-hold differentiator ξ_{mr} is compared to that using the backward differentiator ξ_{bd}. The feedforward controller with acceleration and snap is used in the experiment. The same viscous friction compensation with the basis function $\dot{r}[k] = \mathcal{S}\frac{\mathrm{d}}{\mathrm{d}t}r(t)$ is used in each method. The tuning parameters are optimized by the norm-optimal iterative learning control with several iterative experiments (Bolder et al., 2014).

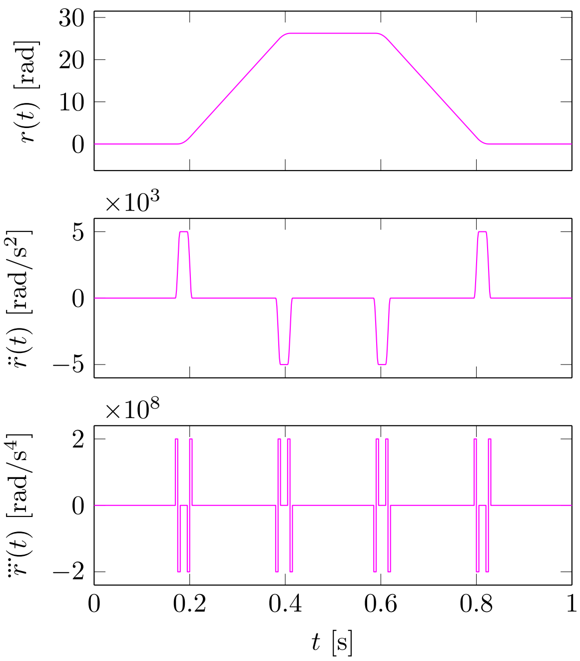

Fig. 7. Continuous-time 4th order polynomial trajectory reference: position $r(t)$, acceleration $\ddot{r}(t)$, and snap $\ddddot{r}(t)$.

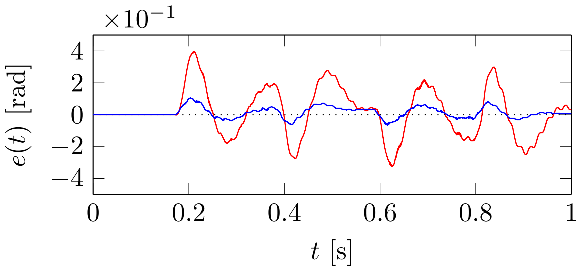

Fig. 8. Tracking error $e(t)$ in experiment with $\boldsymbol{\Psi} = [\dot{r}, \Psi_a, \Psi_s]$: using backward differentiator (—) and multirate zero-order-hold differentiator (—). The feedforward using the multirate zero-order-hold differentiator outperforms that using the backward differentiator in both rigid and flexible modes.

4.3 Experimental validation

The continuous-time error $e(t)$ in the experimental result is shown in Fig. 8. The tracking performance using the multirate zero-order-hold differentiator is improved in two points compared to that using the backward differentiator. First, the acceleration feedforward compensates for rigid dynamics correctly and the error during acceleration and deacceleration periods of the reference becomes smaller. Second, the snap feedforward compensates flexible dynamics correctly and the error because of the mechanical resonance becomes smaller. The results demonstrate that the feedforward using the multirate zero-order-hold differentiator outperforms that using the backward differentiator in both rigid and flexible modes.

5. CONCLUSION

The low-order feedforward control approach considering the sampled-data characteristics is developed to improve both on-sample and intersample performance for reference tracking in multi-modal motion systems. The feedforward controller is linearly parameterized and the basis functions are designed using the multirate zero-order-hold differentiator for the state compatibility to the continuous-time reference. Application to the multi-modal motion system demonstrates a significant improvement in tracking performance compared to the conventional approach in the experiment. Ongoing research focuses on learning the tuning parameters from the experimental data, rational basis functions considering the sampled-data characteristics, and extension to the multi-input multi-output systems.

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