Basic Idea of Quadrant Dynamic Programming for Adaptive Cruise Control to Create Energy Efficient Velocity Trajectory of Electric Vehicle

Mitsuhiro Hattori
Graduate School of Frontier Sciences
The University of Tokyo
Kashiwa, Japan
hattori.mitsuhiro18@ae.k.u-tokyo.ac.jp

Hiroshi Fujimoto
Graduate School of Frontier Sciences
The University of Tokyo
Kashiwa, Japan
fujimoto@k.u-tokyo.ac.jp

***Abstract*—Previous studies proposed various optimization algorithms such as gradient method and model predictive control (MPC) to reduce the energy consumption of vehicles with adaptive cruise control. Reducing energy consumption is achieved by optimal velocity control and reducing energy loss. We propose an approach based on dynamic programming (DP). DP is a feedback control with a calculated table of inputs. Autonomous driving trains widely use this method for reducing energy consumption. We created an algorithm, quadrant dynamic programming (QDP), to calculate optimal velocity trajectory. We divided the table into quadrants and seamlessly connected them. With this algorithm, we managed to support many situations even though the table is two-dimension. The result of the simulation and bench tests with an actual vehicle support the fact that the algorithm is valid.**

***Index Terms*—electric vehicle, dynamic programming, energy efficiency, optimization, adaptive cruise control**

I. Introduction

Adaptive cruise control (ACC) was developed to maintain a safe, preset minimum distance between cars in the same lane [1]. Since ACC enables autonomous car following without driver's input, ACC creates more energy-efficient driving experience [2]. Energy-efficient drive technologies are essential for three reasons, range extension, CO_2 emission, and running cost. Range extension is especially important for electric vehicles, whose cruising distance per supply is relatively shorter than gasoline vehicles [3]. From environmental aspects, there are possibilities to reduce CO_2 emissions significantly by energy-efficient drive, because the transportation sector contributes 24% of the world's CO_2 emissions from fuel combustion [4].

Furthermore, from an economic point of view, the energy efficiency of the vehicle leads to a reduction of running cost, which is getting more dominant with sharing economy [5]. Achieving energy-efficient driving is, therefore, getting more critical and energy efficiency of electric vehicles is widely studied [6]. In this paper, energy-efficient driving is achieved by optimal velocity control and reducing energy loss.

Energy loss of electric vehicles consists of electrical loss and dynamical loss. Motor's copper loss and iron loss contribute to the electrical loss, and dragging forces such as air resistance, viscous resistance, and rolling friction cause dynamical loss. In this paper, we reduce all the loss together by achieving optimal acceleration and deceleration. Previous studies proposed various optimal control techniques to reduce the energy consumption of the vehicle by autonomous driving.

(a) Experimental vehicle.

(b) Simulation bench.

Fig. 1. Picture of FPEV2-Kanon, an experimental vehicle manufactured by our group. In this paper, we use FPEV2-Kanon in experiment and simulation. (a) Picture of FPEV2-Kanon. (b) Simulation bench RC-S of Ono Sokki Co., Ltd.

The gradient method is an optimization method with precomputation. It calculates the optimal input before driving and applies feedforward control with it [7]. This method needs consideration for vehicle control, which has a significant disturbance of the environment.

Model predictive control (MPC) is an online optimization control which predicts a future state. MPC is widely used to reduce the energy consumption of vehicles [8], [9]. In MPC, it is challenging to complete the computation in a step period because of the complexity of the optimization. To solve this problem, methods such as considering constraints with reference governor [10], and combining with the precomputation of DP [11] are proposed. However, it is still challenging to tune parameters of the cost function for MPC.

In this paper, we propose an approach based on dynamic programming (DP). DP is a feedback control with a calculated table of inputs. Autonomous driving trains widely use this method for reducing energy consumption [12], [13]. For electric vehicles, it is also used to calculate optimal velocity

TABLE I
PARAMETERS OF THE IN-WHEEL MOTOR AND THE VEHICLE.

Parameter	Description	Value
J_ω	Wheel inertia	$1.26\,\mathrm{kgm^2}$
L_q	q-axis inductance	$2.34\,\mathrm{mH}$
Φ	Leakage flux	$0.249\,\mathrm{Wb}$
R	Copper resistance	$0.1036\,\Omega$
R_c0	Equivalent iron loss resistance	$454.23\,\Omega$
R_c1	Equivalent iron loss resistance	$0.1516\,\Omega$
K_t	Motor constant	$1.245\,\mathrm{Nm/A}$
P_n	Number of pole pairs	20/2
M	Vehicle mass	$880\,\mathrm{kg}$
g	Gravity acceleration	$9.8\,\mathrm{m/s^2}$
b	Viscous resistance coefficient	$10.7\,\mathrm{kg/s}$
r	Wheel radius	$0.302\,\mathrm{m}$
D_s	Driving stiffness	12
F_a	Air resistance coefficient	$0.552\,\mathrm{Ns^2/m^2}$
μ_r	Rolling resistance coefficient	0.0126
h_g	Height of center of gravity (CG)	$0.51\,\mathrm{m}$
l_f	Distance between CG & front wheel	$1.013\,\mathrm{m}$
l_r	Distance between CG & rear wheel	$0.702\,\mathrm{m}$

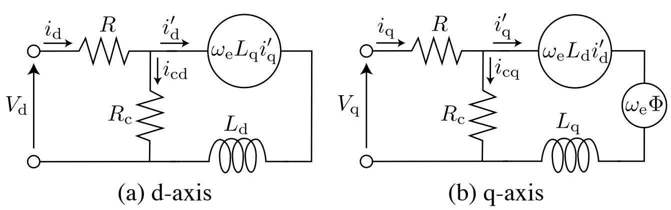


Fig. 2. Equivalent circuit of the permanent magnet synchronous motor (PMSM).

trajectory [14], [15]. DP is a global optimization algorithm and is easy to handle constraints. The result of DP is always best, but the size of the calculated table is too large to support the various situation of the vehicle.

To solve this problem, we created an algorithm of ACC with two-dimension DP instead of three-dimension. The proposed quadrant dynamic programming (QDP) creates a table for four segments, and the situations supported by the algorithm are much more than normal two-dimension DP.

II. EXPERIMENTAL VEHICLE

In this section, the modeling of the experimental vehicle is described. FPEV-2 Kanon, shown in Fig. 1 (a), is an experimental vehicle manufactured by our research group. This vehicle has four independently driven in-wheel motors, and the motor is an outer-rotor permanent magnet synchronous motor (PMSM). In this paper, we use two rear motors as driving wheels.

Since all in-wheel motors are direct-drive, the reaction force transfers to roads without the influence of gear backlash or shaft torsion. Therefore, we only consider the motor's loss and driving resistance as energy loss.

A. Motor's Power Model

The equivalent circuit of the motor is shown in Fig. 2. The input power of the motor P_in is defined as the sum of output

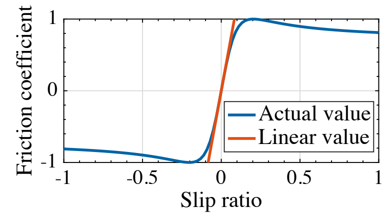


Fig. 3. The relation between slip ratio and road friction coefficient. The magic formula [16] calculates the actual value. Linear value is calculated with driving stiffness.

power P_out, copper loss P_c, and iron loss P_i. Each of them is

$$P_\mathrm{out} = \sum_{\mathrm{all}} \omega T, \quad P_\mathrm{c} = \sum_{\mathrm{all}} R\left(\frac{T}{K_\mathrm{t}}\right)^2,$$
$$P_\mathrm{i} = \sum_{\mathrm{all}} \frac{\omega P_\mathrm{n}}{R_\mathrm{c}} \left\{ \left(L_\mathrm{q}\frac{T}{K_\mathrm{t}}\right)^2 + \Phi^2 \right\}, \tag{1}$$

where T and ω are torque and rotational speed, and equivalent iron loss resistance R_c is

$$\frac{1}{R_\mathrm{c}} = \frac{1}{R_\mathrm{c0}} + \frac{1}{R_\mathrm{c1}|\omega P_\mathrm{n}|}. \tag{2}$$

Parameters are shown in Table I.

B. Vehicle Dynamics Model

The dragging force of the vehicle is described as

$$F_\mathrm{DR} = \mu_\mathrm{r} M g + b|v| + F_\mathrm{a} v^2, \tag{3}$$

where each term refers to rolling resistance, viscous resistance, and air resistance. Each parameter is shown in Table I.

To calculate driving force, slip ratio λ is defined as

$$\lambda = \frac{r\omega - v}{\max(v, r\omega)}, \tag{4}$$

where ω is wheel angular velocity. Relation between road friction coefficient μ and slip ratio λ is shown in Fig. 3.

In normal driving situation of $|\lambda| \ll 1$, friction coefficient is considered as linear and calculated as $\mu = D_\mathrm{s}\lambda$, where D_s is the normalized driving stiffness. With this relation, we calculate the driving force of each wheel as $F = \mu N = D_\mathrm{s}\lambda N$, where N is the load force of the tire.

The equation of rotational motion and equation of the vehicle motion are

$$J_\omega \dot{\omega} = T - rF, \quad M\dot{v} = \sum F - F_\mathrm{DR}, \tag{5}$$

where T is torque and F is the driving force of each wheel.

III. PROBLEM FORMULATION

In this section, the formulation of the optimization problem is described. The objective of the optimization is to minimize the energy consumption of the electric vehicle in a vehicle-following situation of ACC. As shown in Fig. 4, the controlled vehicle aims to drive the same speed as the preceding vehicle and to keep adequate distance to it.

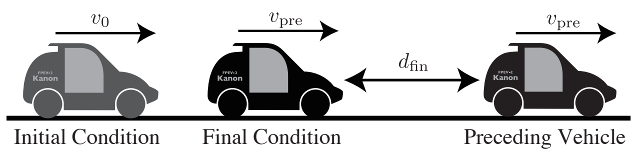


Fig. 4. Illustration of problem formulation of ACC. The controlled vehicle aims to drive the same speed as the preceding vehicle and to keep adequate distance to it.

Since dynamic programming divides the problem into multiple subproblems, the problem is discretized by velocity and distance. Thus the model is approximated to be calculated by the discretized velocity.

A. Approximated Model

To apply dynamic programming (DP), we calculated the motor power with velocity instead of toque and rotation speed. The driving force F_{r}, load force, and slip ratio λ_{r} of one rear tire is calculated as

$$F_{\mathrm{r}} = \frac{1}{2}F_{\mathrm{all}} = \frac{1}{2}\left(M\dot{v} + \mu_{\mathrm{r}} Mg + b|v| + F_{\mathrm{a}} v^2\right),$$
$$N_{\mathrm{r}} = \frac{1}{2}\left(\frac{l_{\mathrm{f}}}{l}Mg + \frac{h_{\mathrm{g}}}{l}M\dot{v}\right), \quad \lambda_{\mathrm{r}} = \frac{F_{\mathrm{r}}}{D_{\mathrm{s}} N_{\mathrm{r}}}. \tag{6}$$

With these values, torque and rotation speed is calculated as

$$T_{\mathrm{r}} = rF_{\mathrm{r}} + J_{\omega}\dot{v}\frac{1+\lambda_{\mathrm{r}}}{r}, \quad \omega_{\mathrm{r}} = \dot{v}\frac{1+\lambda_{\mathrm{r}}}{r}. \tag{7}$$

The motor power is calculated by substituting these values for Eq. (1).

B. Optimization Problem

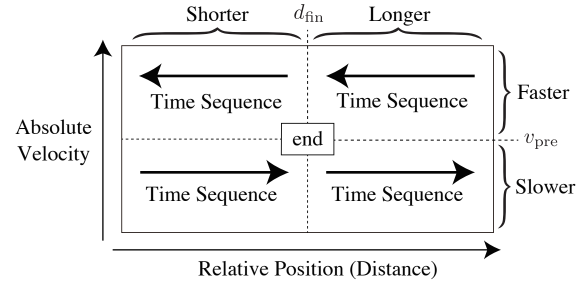
In this paper, we propose a minimization of energy consumption. The velocity trajectory is optimized for multiple velocities of the preceding vehicle. For a velocity of preceding vehicle v_{pre}, the optimization problem is formulated as

$$\min \ J = \sum_{i=0}^{n} P_{\mathrm{in}}(v_i, \dot{v}_i)\ t_i, \tag{8}$$

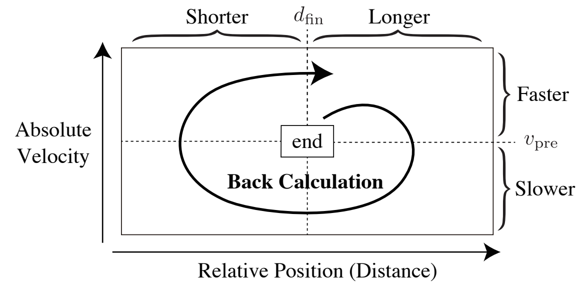
subject to

$$t_i = \Delta d/(v_i - v_{\mathrm{pre}}), \quad \dot{v}_i = \Delta v/t_i, \tag{9}$$
$$F_i = F_r(v_i, \dot{v}_i), \quad |F_i| \le F_{\mathrm{max}}, \tag{10}$$
$$d_{i+1} = d_i + \mathrm{sgn}(v_i - v_{\mathrm{pre}})\Delta d, \tag{11}$$
$$v_n = v_{\mathrm{pre}}, \quad 0 \le v_i \le v_{\mathrm{max}}, \tag{12}$$
$$d_n = d_{\mathrm{fin}}, \quad 0 \le d_i \le d_{\mathrm{max}}, \tag{13}$$

where Δd and Δv are mesh size of discretized distance and velocity. Eq. (9) is the actual calculation of $\dot{v}$ and t_i. Eq. (10) is a constraint for tire force and Eq. (11) is an update of distance mesh. Eq. (12) and Eq. (13) are the final condition and constraint for velocity and distance.



(a) Time sequence of each segment.



(b) Visualization of back calculation.

Fig. 5. Basic algorithm of quadrant dynamic programming (QDP). We divided the table into 4 segments to make the time sequence for back calculation. (a) Time sequence is consistent for each segment. (b) Calculation is done by going backwards along the time sequence.

IV. Algorithm

In this section, the proposed algorithm, quadrant dynamic programming (QDP), is described. Dynamic programming (DP) is a commonly used optimization algorithm for trains [13]. DP divides the problem into multiple subproblems, and the optimization is solved efficiently with the results of subproblems.

Normally, the problem is discretized by velocity, distance, and time to apply DP [14]. However, for vehicle following situation, it takes months to calculate the three-dimension DP for multiple velocity patterns of the preceding vehicle. The proposed QDP is two-dimension DP, and the calculation time is a few minutes. Thus the QDP is a more practical algorithm.

While driving, the optimal velocity trajectory for the very situation is created instantaneously by backward search of precomputed table of DP [17].

A. Quadrant Dynamic Programming

To calculate the optimal velocity trajectory, the time sequence of the table needs to be consistent. Thus the table is divided into 4 segments at final distance d_{fin} and at velocity of preceding vehicle d_{pre}. As shown in Fig. 5, each segments have each time sequence. Thus the optimal trajectory is calculated by back-calculation from the final cell.

As shown in Fig. 6, we use the minimum energy consumption for each cell. Calculation in each segment is based on normal DP, but the calculation uses the previously calculated segment to connect segments seamlessly. The cost of the final cell is set to 0, and the cost of other cell is the minimum energy required to reach final state.

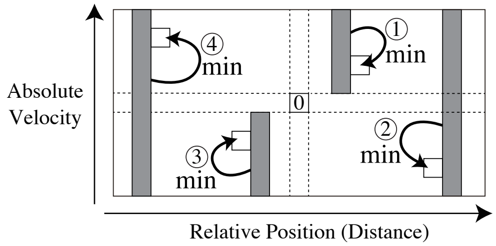


Fig. 6. Calculation of optimization using quadrant dynamic programming (QDP). To connect segments seamlessly, the calculation uses the previously calculated segment.

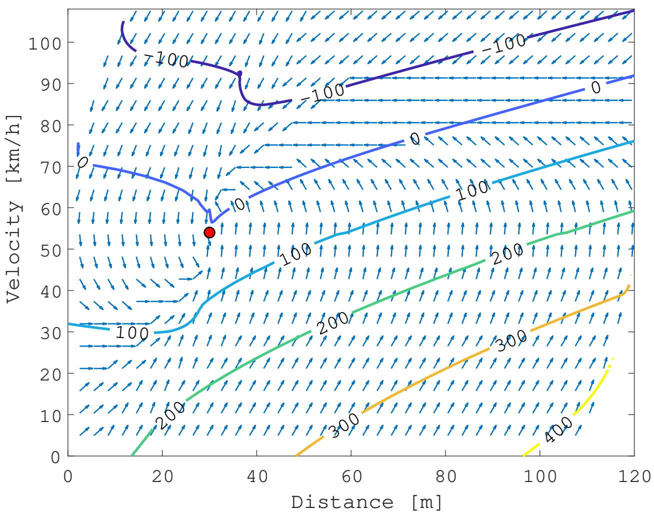


Fig. 7. Optimization result for $v_{\mathrm{pre}} = 54\,\mathrm{km/h}$. The contour shows the required energy to final state [kJ]. From the initial distance and velocity, the optimal velocity trajectory is calculated by following the arrows. 'Velocity' is the absolute velocity of the vehicle and 'Distance' is the relative position of the two vehicles.

Each cell has information of the next cell and the cost. The optimal trajectory is created by tracing the information of the next cell from the initial cell.

B. Optimization Result

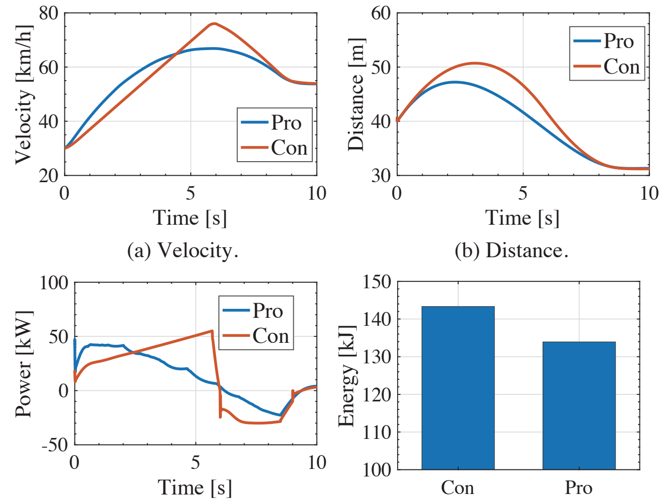
Optimization result for $v_{\mathrm{pre}} = 54\,\mathrm{km/h}$ is shown in Fig. 7. The contour shows the cost to the final cell, which is the required energy to the final state. The arrows show the information of the next cell. Tracing the arrows creates the optimal velocity trajectory. In this case, the final distance d_{fin} is set to $30\,\mathrm{m}$.

V. Evaluation

In this section, the results of the proposed method are shown. We conducted simulations and a bench test.

A. Simulation

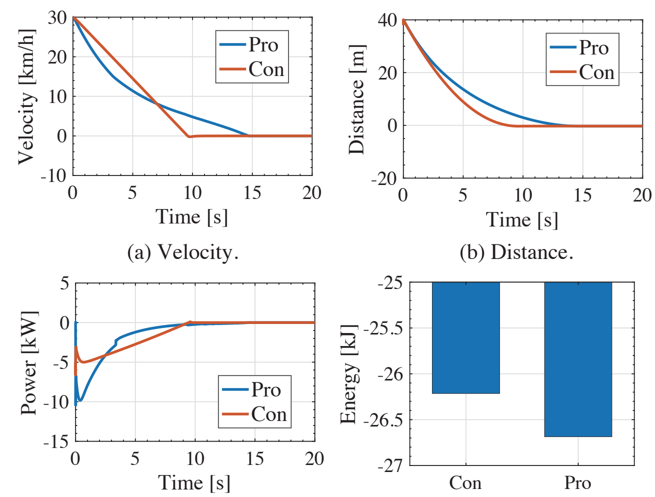
Simulation is conducted to evaluate the result of optimization. We did the simulation with a full model of our experimental vehicle using MATLAB Simulink. The vehicle is controlled with velocity controller to follow the calculated velocity trajectory.



(a) Velocity. (b) Distance.

(c) Power. (d) Energy.

Fig. 8. Simulation result for $v_{\mathrm{pre}} = 54\,\mathrm{km/h}$. Initial velocity and distance are $30\,\mathrm{km/h}$ and $40\,\mathrm{m}$. Compared with constant acceleration, proposed method reduced energy consumption. 'Velocity' is the absolute velocity of the vehicle and 'Distance' is the relative position of the two vehicles.

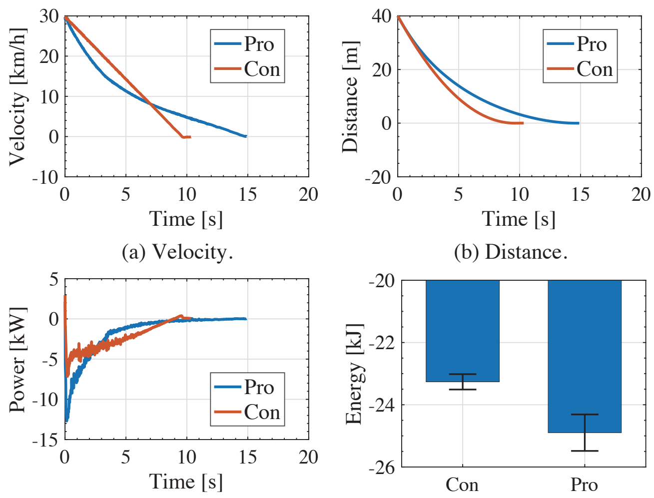


(a) Velocity. (b) Distance.

(c) Power. (d) Energy.

Fig. 9. Simulation result for a preceding vehicle of velocity $v_{\mathrm{pre}} = 0\,\mathrm{km/h}$. Initial velocity and distance are $30\,\mathrm{km/h}$ and $40\,\mathrm{m}$. Compared with constant deceleration, proposed method gained more regenerative energy. 'Velocity' is the absolute velocity of the vehicle and 'Distance' is the relative position of the two vehicles.

Fig. 8 shows the result of following a vehicle of $54\,\mathrm{km/h}$, and Fig. 9 shows the result of stopping at $40\,\mathrm{m}$ position. The initial velocity and distance are $30\,\mathrm{km/h}$ and $40\,\mathrm{m}$ for both simulation. 'Pro' in Fig. 8–9 shows the result of the proposed method of QDP, and 'Con' shows the result of the comparison method of constant acceleration. The comparison is a velocity controller with constant acceleration. The results show that the proposed method minimize energy consumption and maximize regenerative energy.



(a) Velocity. (b) Distance.

(c) Power. (d) Energy.

Fig. 10. Experimental result of bench test for a preceding vehicle of velocity $v_{\mathrm{pre}} = 0\,\mathrm{km/h}$. Initial velocity and distance are $30\,\mathrm{km/h}$ and $40\,\mathrm{m}$. Compared with constant deceleration, proposed method gained more regenerative energy.

B. Bench Test

We conducted a bench test with RC-S of Ono Sokki Co., Ltd. shown in Fig. 1 (b). Since RC-S absorbs driving force by directly connecting driving wheels to a dynamometer, it is capable of testing electric vehicle with fast reaction [18].

For the experiment with bench test, we used the experimental vehicle, FPEV-2 Kanon. RC-S calculates the velocity of the vehicle, and the position is calculated as the time integration of the velocity. For each condition, we experimented five times and calculated the standard error of energy consumption.

Fig. 10 shows the result of stopping at $40\,\mathrm{m}$ position. The initial velocity and distance are $30\,\mathrm{km/h}$ and $40\,\mathrm{m}$ for both simulation. The conditions are the same as the simulation of Fig. 9. This bench test shows that the proposed method is valid, and the simulation is acceptable.

VI. Conclusion

Previous studies proposed various optimization algorithms for the optimization of energy consumption of vehicles. In this paper, we proposed an approach with quadrant dynamic programming. Since dynamic programming is a global optimization, the result is always the best. We divided the table of DP, and the optimization is done with two-dimension DP.

With simulation and bench tests, results show that the proposed algorithm is valid for an actual electric vehicle. We are planning to do more experiments before the final submission.

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