High-Bandwidth Current Control of PMSM Based on Quasi Multirate Feedforward Contol

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PI feedback control is commonly used as the current control of electric motors. However, its tracking performance becomes worse when the reference value changes steeply. To drive motors at higher speeds, the reference tracking performance needs to be guaranteed. We have proposed a current controller based on the perfect tracking control (PTC) before. It has improved the tracking performance of the current control for motors. However, there is still room for improvement, so a new approach to current control of the permanent magnet synchronous motors (PMSMs) based on the quasi multirate feedforward control is proposed. This method based on the quasi multirate control doubles the tracking point of the reference current waveform compared to the conventional method. In this paper, the theory of the new control method is described at first. Next, the design of the controller is described. Finally, simulations and experimental verifications were conducted to demonstrate the effectiveness of the proposed method.

Keywords: permanent magnet synchronous motor (PMSM), current control, perfect tracking control, quasi multirate feedforward control, tracking performance

1. Introduction

In recent years, the electrification of vehicles such as hybrid vehicles (HVs) and electric vehicles (EVs) is trending to prevent air pollution and global warming. Furthermore, some countries have declared the prohibition of selling gasoline and diesel engine cars [1]. Thus, it is expected that the number of electrified vehicles will increase in the future.

In general, the proportional integral (PI) feedback control is used as the current control of motors. However, the problem is that when the reference value is steeply changed, the output current cannot follow it. Therefore, it is necessary to guarantee the tracking performance of the reference value to drive motors at high speeds. Some papers reported experimental verifications of motors for real EVs. For example, it is reported that the model predictive control has improved the tracking performance of the torque reference better than Field Oriented Control (PI control) [2]. Another study analyzed the noise and the vibration of motors for EVs [3]. However, the tracking performance of the torque reference shown in the former study can be improved by the use of the perfect tracking control (PTC) for the current control [4], and the latter one shows that the current control that has the tracking capability up to the carrier frequency to remove the harmonics is necessary. To solve those issues, a new current control method based on PTC is introduced in this paper.

2. Proposed Control Method

2.1 Perfect Tracking Control (PTC)

PTC is the control system that follows the reference value without error at every sampling point [5]. There are two samplers for the reference $r(t)$ and the output signal $y(t)$, and one holder on the input $u(t)$, in a digital control system. Therefore, period parameters are notated as T_r, T_y and T_u, which represent the periods of $r(t)$, $y(t)$, and $u(t)$, respectively.

2.2 PWM Hold model

The single phase inverter (Fig. 1) outputs either 0 V or $\pm E$ V (Fig. 2). For the discretization of this system, a zero-order hold (ZOH) is commonly used. The ZOH outputs $V[k]$ at a sampling point k as the discretized value. However, a more exact plant model can be obtained by treating it as a hold that the width of the pulse is controlled instead of a zero-order hold. To achieve this, a PWM hold is introduced [7]. The control system is discretized with the switching time $\Delta T[k]$ as a control input $u[k]$. The state space model with controllable canonical form (1) and (2) can be described by the PWM hold,

$$\boldsymbol{x}[k+1] = \boldsymbol{A}_s \boldsymbol{x}[k] + \boldsymbol{b}_s \Delta T[k] \tag{1}$$

$$y[k] = \boldsymbol{c}_s \boldsymbol{x}[k], \tag{2}$$

where $\boldsymbol{A}_s = e^{\boldsymbol{A}_c T_u}$, $\boldsymbol{b}_s = e^{\boldsymbol{A}_c T_u/2} \Delta T \boldsymbol{b} E$, $\boldsymbol{c}_s = \boldsymbol{c}_c$ and if $\Delta T < 0$, then the output voltage will be $-E$.

2.3 Multirate Feedforward Control

To achieve PTC, a multirate feedforward control is presented [6]. The state equation is described up to n samples ahead by

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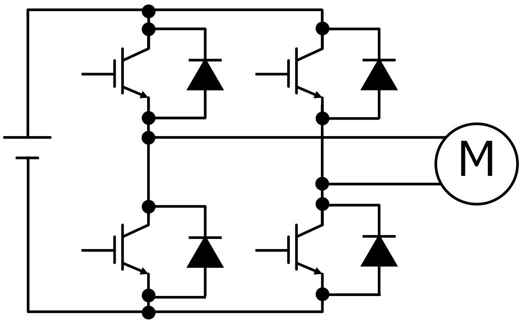


Fig. 1. Circuit Diagram of Single Phase Inverter

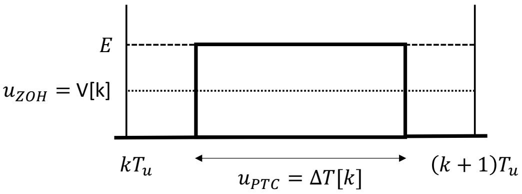


Fig. 2. DC Voltage Waveform Discretized by PWM Hold

lifting. Here, n is the plant order. The controllable canonical state equation (1)–(2) is discretized with the sampling period T_u of the multirate holder. The discretized $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ matrices of the control system are

$$\left(\begin{array}{c|c} \boldsymbol{A} & \boldsymbol{B} \\ \hline \boldsymbol{C} & \boldsymbol{D} \end{array}\right) = \left(\begin{array}{c|cccc} \boldsymbol{A}_s^n & \boldsymbol{A}_s^{n-1}\boldsymbol{b}_s & \cdots & \boldsymbol{A}_s\boldsymbol{b}_s & \boldsymbol{b}_s \\ \hline \boldsymbol{c}_s & 0 & \cdots & 0 & 0 \\ \boldsymbol{c}_s\boldsymbol{A}_s & \boldsymbol{c}_s\boldsymbol{b}_s & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \boldsymbol{c}_s\boldsymbol{A}_s^{n-1} & \boldsymbol{c}_s\boldsymbol{A}_s^{n-2}\boldsymbol{b}_s & \cdots & \boldsymbol{c}_s\boldsymbol{b}_s & 0 \end{array}\right). \tag{3}$$

The controllability matrix is full rank since the system is controllable, so $\boldsymbol{B}$ matrix is a holomorphic matrix, then (3) can be rewritten as

$$u_o[k] = \boldsymbol{B}^{-1}(\boldsymbol{I} - z^{-1}\boldsymbol{A})\boldsymbol{x}[k+1] \tag{4}$$

$$= \left(\begin{array}{c|c} 0 & \boldsymbol{I} \\ \hline -\boldsymbol{B}^{-1}\boldsymbol{A} & \boldsymbol{B}^{-1} \end{array}\right)\boldsymbol{x}[l+1], \tag{5}$$

$$y_o[l] = z^{-1}\boldsymbol{C}\boldsymbol{x}[l+1] + \boldsymbol{D}u[l], \tag{6}$$

where, $z = e^{sT_r}$.
Therefore, the nominal output that is a stable inverse model of the plant is described. The inverse system that uses the state variable of one sample ahead $\boldsymbol{x}[l+1]$ as the input variable can be calculated by treating with the multirate conversion. This guarantees the perfect tracking of the nominal plant at every period T_r.

2.4 Quasi Multirate Control

The quasi multirate PTC is proposed in this subsection. Fig. 4 shows the principle of the quasi multirate control. The pulse width is generated between the sampling points using the state variable k corresponding to the control period. In the second order multirate control system, the first and second lines of the (4) are

$$u[k+1] = \boldsymbol{A}_s x[k] + \boldsymbol{b}_s u[k], \tag{7}$$

$$u[k+2] = \boldsymbol{A}_s x[k+1] + \boldsymbol{b}_s u[k+1]. \tag{8}$$

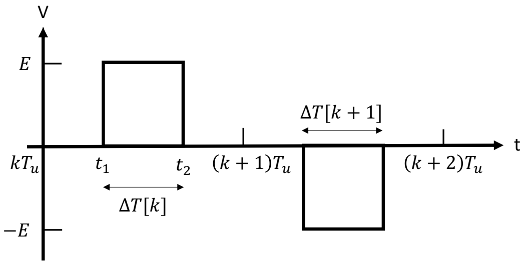


Fig. 3. Output of Single Phase Inverter

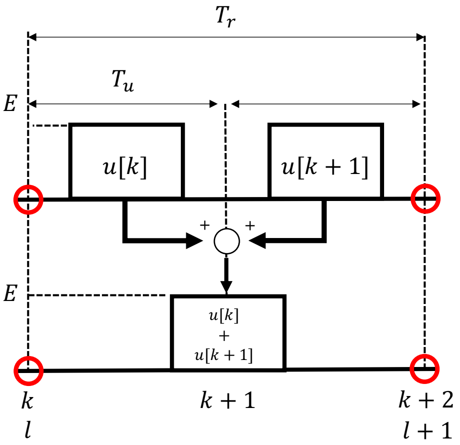


Fig. 4. Principle of Quasi Multirate Control

From (7) and (8), the equation at the next discretization point of the reference value $(l+1)T_r$

$$u[k+2] = u[k+1] + u[k]. \tag{9}$$

Thus, a PWM pulse can be made into the one pulse by adding two pulse widths, and the newly derived $u[k+2]$ is outputted to the plant [9]. The one pulse can be obtained as shown in Fig. 4. As a result, the number of output pulses of the quasi multirate PTC is smaller than that of the multirate PTC, so that the switching loss can be reduced while guaranteeing the reference value tracking at every sampling point. This is because the average output voltage between kT_u and $(k+2)T_u$ is as same as before combining the pulses. For this reason, the number of the switching can be doubled. This method is shown in Fig. 5. The new sampling period of the reference signal T_r' is defined as $T_r/2$ and feedforward calculation is held every T_u while the pulses are enhanced at T_r' that equals to the carrier frequency. Therefore, with keeping zero-tracking error at every T_r', the proposed multirate feedforward control can be held.

3. Simulation

3.1 Plant Modeling

In this study, SPMSM is a plant, and its dq coordinate model is shown in Fig. 6. Now, if the d-axis current $i_d = 0$ is maintained by the current controller and non-interference control, the plant model on the q-axis can be assumed to be a DC motor as shown in Fig. 7 This is expressed in the same way as a transfer function from the voltage input to current output in the continuous time in this model is obtained. Since the transfer function from the voltage input to the current output must include the feedback loop due to the effect of the reverse

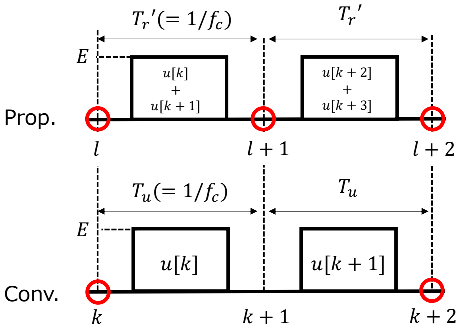


Fig. 5. Comparison of PWM Pulse Pattern between Quasi Multirate and Multirate Control

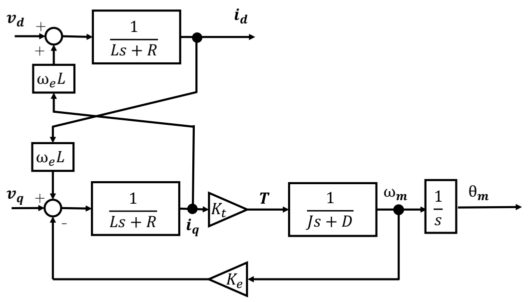


Fig. 6. Block Diagram of SPMSM at dq-axis with Decoupling Control

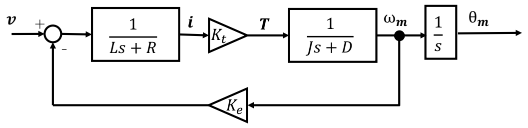


Fig. 7. Block Diagram of DC Motor

voltage, the plant model would be second order plant model.

$$\frac{i}{v} = \frac{Js+B}{JLs^2 + (JR+LB)s + (BR+K_eK_t)}. \tag{10}$$

The controllable canonical form of this plant is described as follows, with the state variable $\boldsymbol{x}(t)$ as the current and the differential of that $[i\ \dot{i}]^{\mathrm{T}}$

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_c\boldsymbol{x}(t) + \boldsymbol{b}_c u(t), \tag{11}$$

$$y(t) = \boldsymbol{C}_c\boldsymbol{x}(t) \tag{12}$$

where,

$$\boldsymbol{A}_c = \begin{bmatrix} 0 & 1 \\ -\frac{BR+K_eK_t}{JL} & -\frac{JR+LB}{JL} \end{bmatrix}, \tag{13}$$

$$\boldsymbol{b}_c = [0\ \frac{1}{JL}]^{\mathrm{T}}, \tag{14}$$

$$\boldsymbol{c}_c = [B\ J]. \tag{15}$$

As described in the equations above, the simulation is performed by applying the multirate PTC and the quasi multirate PTC to the plant model (10).

3.2 Controller Design

The plant model has a feedback loop due to the reverse voltage, so the multirate controllers are applied. $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ matrices of the PTC controller are determined by the dq-axis model of the SPMSM. The current controller C_{PI} is designed as a pole-zero canceled PI feedback controller as follows.

Table 1. Parameters of the motor for the simulation

Parameter	Value
Time Constant τ [ms]	1.0
Resistance $R[\Omega]$	0.1567
Inductans L [mH]	2.28
Inertia J [kg · m^2]	9.084×10^{-4}
Viscous Resistance D [N · m · s/rad]	4.00×10^{-4}
Pole Pairs P	4
Induced Electromotive Voltage K_e [V · s/rad]	0.1727
Torque Constant K_t [N · m/A]	0.1727
Carrier Frequency f_c [kHz]	10
Control Input Period T_u [s]	1.00×10^{-4}
DC Input Voltage of Inverter E [V]	250
Reference Current Input $I_{q\mathrm{ref}}$ [A]	1
Reference Current Frequency f_{ref} [kHz]	2.5

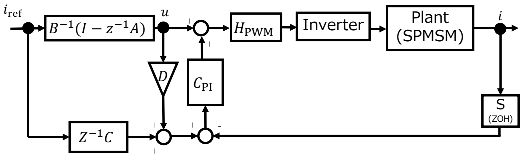


Fig. 8. Current Control based on PTC

$$C_{\mathrm{PI}}(s) = \frac{Ls+R}{\tau s}. \tag{16}$$

Thus, the transfer function from i_{ref} to i is seen as the first-order system as follows.

$$\frac{i}{i_{\mathrm{ref}}} = \frac{1}{\tau s + 1}. \tag{17}$$

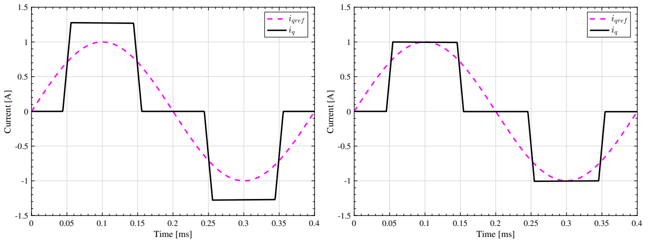
Furthermore, to design the feedforward controller, the transfer function from the voltage input to the current output is described as (10). This is discretized by the PWM hold. The controllable canonical state equation is described by the transfer function of the plant (10) and then is substituted for (1)–(2) to determine the parameters in the discrete-time. The parameters are substituted for the equation (3)–(6) to determine $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{D}$ matrices of the multirate feedforward controller.

3.3 Simulation

Fig.8 shows the block diagram of the proposed method. Table. 1 shows the parameters of the simulation. The frequency of the reference current is 2.5 kHz ($=f_c/4$) in the simulation. The simulation results in Fig. 9 shows the current waveform of the multirate PTC (a) and quasi multirate PTC (b) respectively. From Fig. 9, PTC can follow the reference value and the proposed method is more accurate than the multirate PTC in Fig. 9(a). Moreover, the error between the input and output current is shown in Fig. 10. From Fig. 9(b), the proposed method is a more accurate waveform compared with the multirate PTC. Furthermore, Fig. 10(b) shows that the proposed method guarantees the performance of reference value tracking twice as much as the multirate PTC. Therefore, the proposed method can follow the reference waveform more accurately.

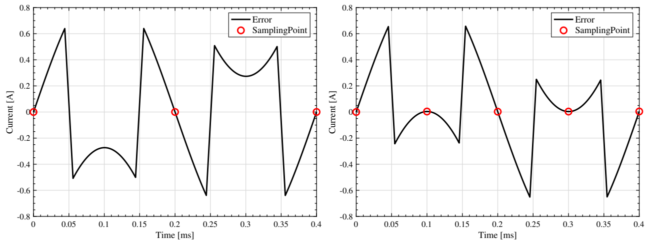
4. Experiment

Fig. 11 shows the motor bench that is used in the current control. The results of the current wave are shown in Fig. 13 and 12. The PI control and the multirate PTC are demonstrated as the conventional methods, and the quasi multirate



(a) Multirate PTC (b) Quasi Multirate PTC

Fig. 9. Current Waveform



(a) Multirate PTC (b) Quasi Multirate PTC

Fig. 10. Error between Current Input and Output

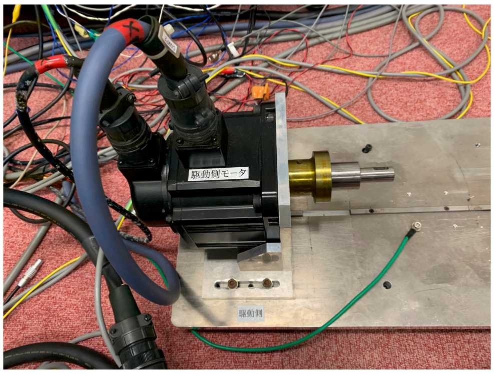
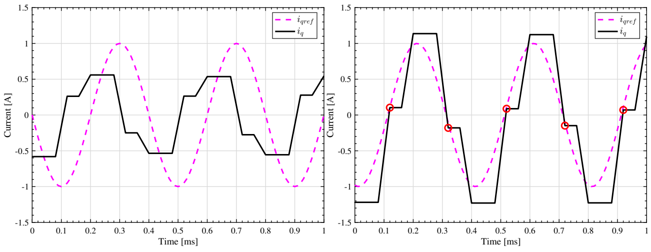


Fig. 11. Motor Bench (SPMSM)



(a) PI Control (b) Multirate PTC

Fig. 12. Current Waveform (Conventional Methods)

PTC is the proposed method. From the results, the proposed method successfully demonstrated two times better reference tracking performance than the conventional methods, as the current output matches roughly every 0.1 ms (twice more red circles than the proposed method). It guarantees to follow the reference twice better than the conventional methods.

5. Conclusion

A new current control method based on the quasi multirate PTC is proposed in this study. Though our group had previously reported that the multirate PTC controller can improve the output tracking performance compared with PI control, the proposed method improved the tracking performance twice as much as the multirate PTC. The effectiveness of the proposed method has been demonstrated in simulations and experiments. The experiment has been conducted by the motor bench, so it is a future work to demonstrate the effectiveness of the proposed method for electric vehicle motors.

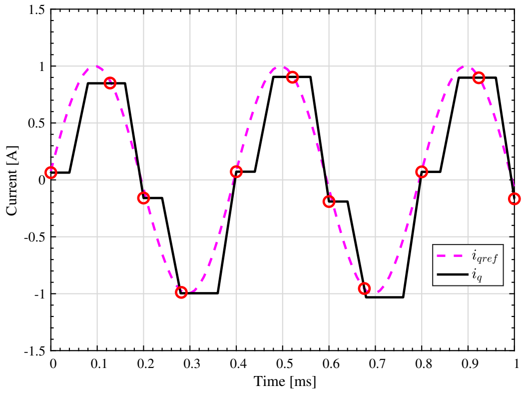


Fig. 13. Current Waveform (Proposed Method)

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