Mutual Inductance Modeling of In-wheel Arc-shaped Coil for In-motion WPT

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***Abstract*—In-motion WPT can solve the problem of electric vehicles that is short cruising range. Therefore transfer energy is one of the biggest problem of In-motion WPT. To prevent contamination is the best way to increase transfer energy for in-motion WPT because of its short time use and long time of contamination detection system. Here, WPT system that can transfer through tire and wheel is proposed and also its coils' mutual inductance model is proposed.**

***Index Terms*—Electric vehicle, In-motion wireless power transfer, In-wheel motor**

I. Introduction

Cruising range is a one of the biggest specific problem of electric vehicles. There is a breakthrough technology to extend cruising range, that is In-motion Wireless Power Transfer(WPT). However WPT system also has problems.

One of the biggest problem is foreign objects which are small animals or metal pieces between transmission coil and receiving coil. If there are small animals or metal pieces, there is risk of damage animals or ignition by eddy current loss. It is dangerous situation and necessary to detect foreign objects and stop transmitting or remove them. WPT charging energy is decided by output, efficiency and charging time. Therefore, in-motion WPT system has to detect the foreign objects much more quickly than WPT system for parking because of the short time from starting WPT to the end of WPT. Equivalent circuit model [1] for detection system is proposed to solve this problem, but the fundamental solution for foreign objects is to prevent them.

II. In-wheel Arc-shaped Coil

To prevent foreign objects, we propose to place the receiving coil inside the wheel, as shown in Fig. 1. In this system, tire and wheel are between transmission coil and receiving coil, foreign objects can not enter there. Conventional receiving coil for wireless in-wheel motor system [2] is outside of wheel, because the metal wheel and the bead wires in the tire is made of metal and they occur eddy current. Especially, metal wheel has not only loss but also is like magnetic shield. Then, wheel made of carbon fiber reinforced plastics (CFRP) is adopted, and there is little effect of magnetic field [3] by wheel. Conventional receiving coil is planer shape for minimizing coil gap. However planer-shaped coil's air gap is bigger, when it is inside of wheel. Therefore we propose arc-shaped coil to minimize air gap for in-wheel motor system. that is shown in Fig. 2.

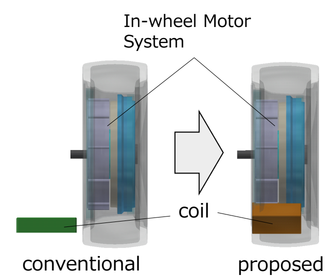

Fig. 1. Coil Position.

III. Mutual Inductance Model

This In-motion WPT system uses series series (SS) system. The equivalent circuit of SS system is shown in Fig. 3.

Here, R is a coil resistance, L is a coil inductance, C is a capacitance of the resonance capacitor, R_{ac} is an equivalent load resistance, L_m is the mutual inductance, and the subscripts 1 and 2 represent the transmission side and the receive side, respectively. ω_0 is the resonance frequency. From Fig. 3,

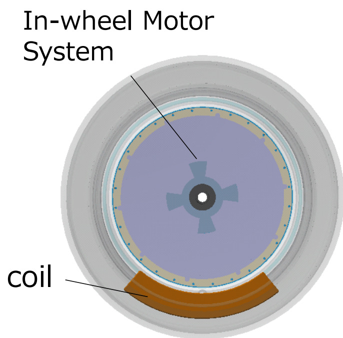

Fig. 2. Arc-shaped Coil.

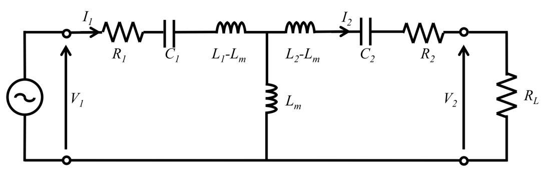

Fig. 3. Equivalent circuit of magnetic resonant coupling.

The transmission power P_{ac},and the transmission efficiency η are respectively, given as follows:

$$P_{ac} = \frac{(\omega_0 L_m)^2 R_{ac}}{(R_1(R_2+R_{ac})+(\omega_0 L_m)^2)^2} V_1^2 \tag{1}$$

$$\eta = \frac{(\omega_0 L_m)^2 R_{ac}}{(R_2+R_{ac})\left(R_1 R_{ac}+R_1 R_2+(\omega_0 L_m)^2\right)} \tag{2}$$

Therefore Mutual inductance is important parameter to calculate transfer power and transfer efficiency.

The rectangular coils are arranged to face each other as shown in Fig. 4., and current flows in coil T, which represents the i-th turn from the inside of the transmission coil Coil R represents the j-th turn from the inside of the receiving coil.

The magnetic flux passing through the coil R caused by the current flowing in a side CD is expressed by the following equation [4].

$$\Phi_{CD} = \frac{\mu_0}{2\pi}\Big[R_{CA'} - (a+c)\text{arctanh}\frac{a+c}{R_{CA'}} - R_{CD'} + (a-c)\text{arctanh}\frac{a-c}{R_{CD'}} - R_{CB'} + (a+c)\text{arctanh}\frac{a+c}{R_{CB'}} + R_{CC'} - (a-c)\text{arctanh}\frac{a-c}{R_{CC'}}\Big] \tag{3}$$

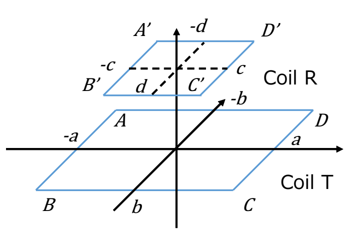

Fig. 4. Rectangular Model.

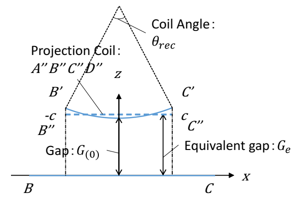

Fig. 5. Arc Shaped Model.

where $R_{AA'}$ represents the distance between points A and A'. The mutual inductance of the coil T and coil R is

$$M_{ij} = \Phi_{AB} + \Phi_{BC} + \Phi_{CD} + \Phi_{DA}. \tag{4}$$

The total mutual inductance is taken as the sum of

$$M = \sum_{i=1}^{N^T} \sum_{j=1}^{N^R} M_{ij} \tag{5}$$

Air gap between arc shaped coil and rectangular coil is not constant. Therefore equivalent gap is proposed. Equivalent gap is shown in Fig. 5. and expressed by the following equation.

$$G_e = \frac{1}{R_{B''C''}} \int_{-c}^{c} G(x)dx \tag{6}$$

The arc shaped model is considered as projected model from transfer side. Therefore coil pitch is not constant. Coil pitch is expressed by the following equation.

$$P_j = R_{coil}(\cos(\frac{\theta_{rec}}{2}) - \cos(\frac{\theta_{rec}}{2} - \theta_{pitch}(j-1))) \tag{7}$$

where θ_{rec} is coil angle, and θ_{pitch} is coil pitch angle. P_j is projection j-th coil pitch which is j-th turn from the outside of the receiving coil. R_{coil} is radius of the coil.

The coordinates of the coil can be calculated by Equation(6) and Equation(7). Finally mutual inductance is calculated with these coordinates and Equation(3), (4), (5).

IV. Mutual Inductance Evaluation of Small Model

The validation in this calculation model is verified with one third scale model. Parameters of evaluation model are shown in Table. I. There are 3 receiving coils for evaluations, rectangular coil and 2 arc-shaped coils. Difference between 2 arc-shaped coils is radius R_{coil}.

Evaluation result is shown in Fig. 6. The error between calculation and actual measurement is $\pm 5\%$.

Comparison between rectangular coil and arc-shaped coil is shown in Fig. 7. with this calculating model. The air gap of rectangular coil is 4.7mm bigger than arc-shaped coil by constraint to set the coil inside of 98.5mm radius cylinder. As the result, mutual inductance of arc-shaped coil is bigger than rectangular coil's.

TABLE I
PARAMETER OF EVALUATION MODEL

Items	Transmission	Receive	
Turns	12	13	
Pitch[mm]	1.9	1.9	
Length[mm]	330	60	
Width[mm]	80	60	
Radius[mm]	-	50.5	98.5

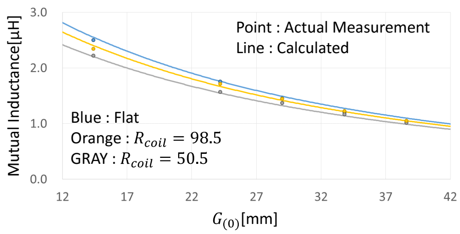

Fig. 6. Evaluation Result of Calculation Model.

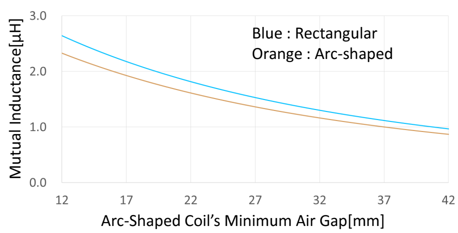

Fig. 7. Rect Angular vs Arc-Shaped($R_{coil} = 98.5°$).

TABLE II
PARAMETER OF FULL SCALE MODEL

Items	Parameter
Diameter of Tire $D_t[mm]$	644
Diameter of Wheel Rim [inch]	20
Diameter of Wheel Inside D_{wi} [mm]	464
Clearance between Wheel and Coil Case C_w [mm]	5
Thickness of Coil Case T_{c1} T_{c2} [mm]	2
Diameter of Coil D_{c1} D_{c2} [mm]	3
Thickness of Road Pavement Material T_r [mm]	10

V. DESIGN OF FULL SCALE MODEL

The development target size of full scale model is shown in Table.2.

Minimum Coil G_0 of arc-shaped coil's gap is expressed by the following equation.

$$G_0 = \frac{D_t - D_{wi} + D_{c1} + D_{c2}}{2} + C_w + T_{c1} + T_{c2} + T_r \quad (8)$$

where $G(0)$ is minimum air gap of arc shaped coil, and D_t is the diameter of tire radius. D_{wi} is the diameter of wheel inside. C_w is clearance between wheel and coil case that is necessary to prevent the touch, and W_c is thickness of coil case which is made by resin for isolation. Proposed mutual inductance model defines that coil gap is the distance between coil center. Therefore the diameters of coil D_c are considered.

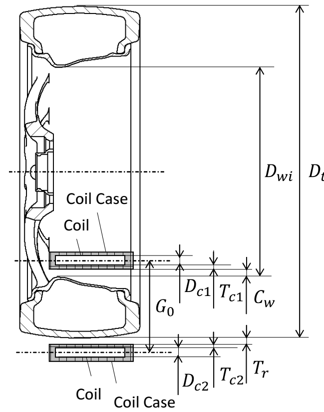

Fig. 8. Minimum Coil Gap

T_r is thickness of road pavement material. Small 1 means transmission side, and 2 means receiving side.

$$G_0 = \frac{D_t - D_{wi} + D_{c1} + D_{c2}}{2} + C_w + T_{c1} + T_{c2} + T_r \quad (9)$$

Items to decide coil gap G_0 are shown in Fig. 8.

Calculating with parameter in Table. II,full scale arc-shaped coil's minimum air gap G_0 is 112mm. To make equivalent gap minimized, it is important to make radius of receiving coil R_{coil} as big as possible, Therefore, R_{coil} is expressed by the following equation, and it is 223.5mm in this model.

$$R_{coil} = \frac{D_{wi} - D_{c2}}{2} - C_w \quad (10)$$

The dimensions of full scale coil are shown in Table. **??**. The total length of arc-shaped coil is set the same as rectangular. Receiving coil has 2 layers to be downsized. Litz wire is adopted for coil to reduce resistance caused by skin effect.

VI. EVALUATION OF FULL SCALE MODEL

Evaluation equipments are shown in Fig. 9. It can adjust the air gap by inserting resin spacers into the air gap. Evaluation results are shown in Fig. 10 and Table. IV when the frequency of transmitting is 85kHz.

The error between calculating model and actual measurement is also ±5%, same as one third model. However the air gap of full scale rectangular coil must be 23.8mm bigger than

TABLE III
DIMENSIONS OF FULL SCALE COIL

Items	Transmission	Received
Outermost Shape of Coil Center[mm]	250*1000	179.5*179.5
Diameter of Coil[mm]	3	3
Coil pitch [mm]	6	6
Number of turns	12	13
Number of Layers	1	2
Outermost Shape of Coil Case[mm]	318*1086*45	230*230*26.5
Wire type	Litz Wire AWG 44*6250	

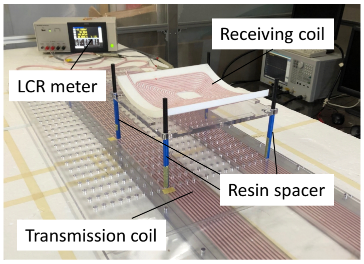

Fig. 9. Evaluation Equipments

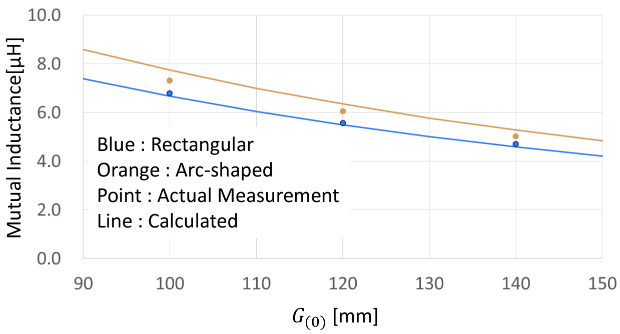

Fig. 10. Gap vs Mutual-inductance

arc-shaped coil's by constraint to set the rectangular coil inside of 225mm cylinder.

Then, the air gap of rectangular coil is 23.8mm bigger than arc-shaped coil's minimum gap to compare with the same condition. Calculated results are shown in Fig. 11 and Fig. 12 considered with constraint of coil position. The horizontal axis is minimum gap of arc shape coil. The voltage of transmission side power supply V_1 is set 300V. The difference between arc the shaped coil and the rectangular coil is caused by bigger air gap.

Actually, there are some metals around receiving coil on

TABLE IV
EXPERIMENTAL COIL PARAMETERS

Items	Rectangular	Arc-shaped
Resistance of Transmission Coil[mΩ]	89.3	
Self-inductance of Transmission Coil[μH]	157.9	
Resistance of Receiving Coil[mΩ]	34.7	34.6
Self-inductance of Receiving Coil[μH]	61.8	60.9

TABLE V
EXPERIMENTAL RESULT WITH 112MM AIR GAP

Items	Values
Resistance of Transmission Coil[mΩ]	249
Self-inductance of Transmission Coil[μH]	152
Resistance of Receiving Coil[mΩ]	35.5
Self-inductance of Receiving Coil[μH]	95.2
Mutual Inductance[μH]	12.1
Maximum Power $V_1 = 300$[V]	6600
Maximum Efficiency[%]	97.7

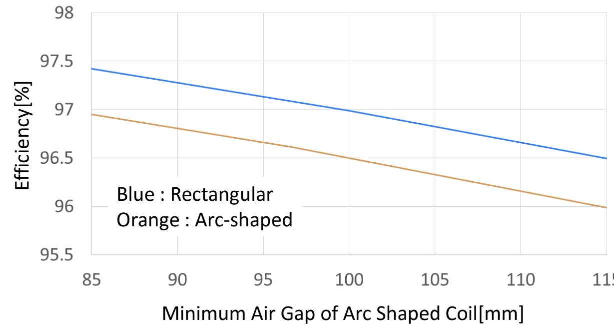

Fig. 11. Gap vs Efficiency(calculated)

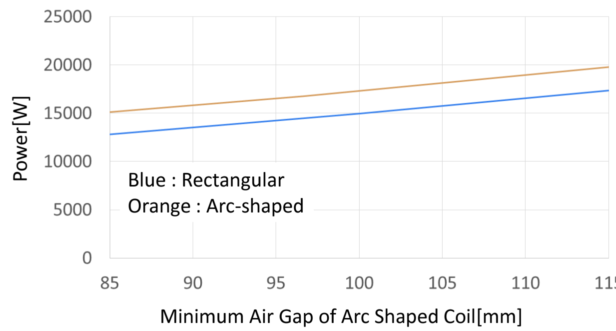

Fig. 12. Gap vs Power(calculated)$V_1 = 300$

the car and it occurs eddy current loss. There are also some regulation about flux leakage. Therefore it is necessary to consider making certain degree of closed magnetic circuit with ferrite. To make closed magnetic circuit, ferrite block is set the back of both of coils. The overview of arc-shaped coil is shown in Fig. 13. The rectangular ferrite block is separated in small pieces and put into arc-shaped ferrite case. It is evaluated with the same experimental equipments as the coil without ferrite.

The evaluation result is shown in Table. V. Maximum power is calculated values when V_1 is 300V. Evaluation results show much increase of both inductance and resistance. They means flux passes ferrite well. As the result, though the maximum output decreases, the maximum efficiency increases. The maximum coil to coil efficiency achieves 97.7 %, moreover the maximum output achieves 6600W per wheel, theoretically. When converted to one car, the maximum output is 13200W(2

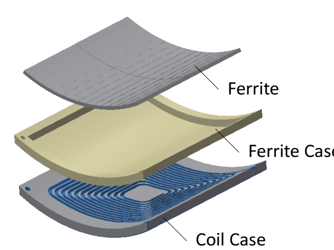

Fig. 13. Overview of Coil and Ferrite

wheels) or 26400W(4 wheels).

VII. Conclusion

In this paper, the novel method for in-motion WPT system, and its mutual inductance model are proposed. The system can reduce possibility to foreign objects entering into between the transmitting coil and the receiving coil. Evaluated model is both small model and full model.

The conclusion is as follows

1) The error between calculating model and actual measurement is $\pm 5\%$, evaluated with small model and full scale model.
2) Arc-shaped coil has more mutual inductance than rectangular coil because of its small air gap.
3) Arc-shaped coil can achieve 97.7% coil to coil efficiency and 6600W output per wheel, theoretically.

The evaluations for efficiency or output by actual WPT are the future work.

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